parameter that the instrument characteristics are sensitive to. Sensitivity drift is measured in units of the form (angular degree/bar)/^{o}C.

A typical change in the output characteristic of a pressure gauge subject to zero drift is shown in Figure 2.3(a). Figure 2.3(b) shows what effect sensitivity drift can have on the output characteristic of an instrument. If an instrument suffers both zero drift and sensitivity drift at the same time, then the typical modification of the output characteristic is shown in Figure 2.3(c).

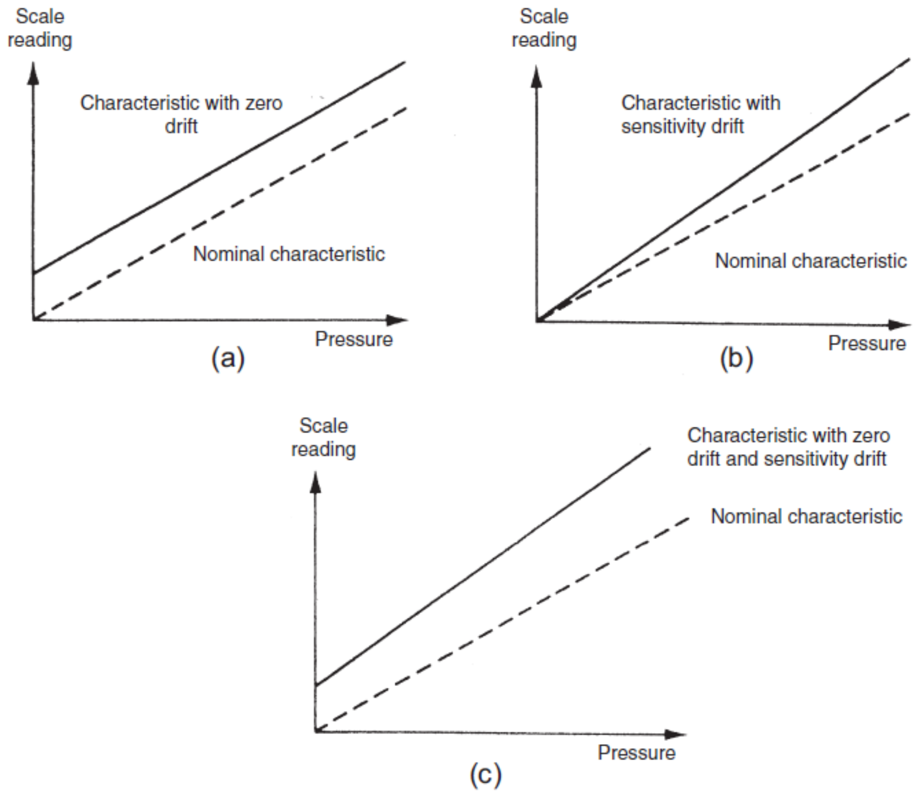

Figure 2.3 Effects of disturbance: (a) Zero drift; (b) sensitivity drift; (c) zero drift plus sensitivity drift.

Example 2.5

The following table shows the output measurements of a voltmeter under two sets of conditions:

(a) Use in an environment kept of 20 ^{o}C, which is the temperature that it was calibrated at and

(b) Use in an environment at a temperature 50 ^{o}C.

Voltage Readings at Calibration Temperature of 20 °C (Assumed Correct)	Voltage Readings at Temperature of 50 °C
10.2	10.5
20.3	20.6
30.7	40.0
40.8	50.1

Determine the zero drift when it is used in the 50 °C environment, assuming that the measurement values when it was used in the 20 °C environment are correct. Also calculate the zero drift coefficient.

Solution

The zero drift at the temperature of 50 °C is the constant difference between the pairs of output readings, i.e., 0.3 V.

The zero drift coefficient is the magnitude of drift (0.3 V) divided by the magnitude of the temperature change causing the drift (30 °C). Thus the zero drift coefficient is 0.3/30 = 0.01 V/°C.

Example 2.6

A spring balance is calibrated in an environment at a temperature of 20 °C and has the following deflection/load characteristic.

Load (kg)	0	1	2	3
Deflection (degrees)	0	20	40	60

It is then used in an environment at a temperature of 30 °C and the following deflection/load characteristic is measured.

Load (kg)	0	1	2	3
Deflection (degrees)	5	27	49	71

Determine the zero drift and sensitivity drift per °C change in ambient temperature.

Solution

At 20 °C, deflection/load characteristic is a straight line. Sensitivity = 20 degrees/kg.

At 30 ^{o}C, deflection/load characteristic is still a straight line. Sensitivity = 22 degrees/ kg.

Zero drift (bias) = 5 degrees (the no-load deflection).

Sensitivity drift = 2 degrees/kg.

Zero drift/^{o}C = 5/10 = 0.5 degrees/^{o}C.

Sensitivity drift/^{o}C = 2/10 = 0.2 (degrees per kg)/^{o}C.

2.2.10 Hysteresis Effects

Figure 2.4 illustrates the output characteristic of an instrument that exhibits hysteresis. If the input measured quantity to the instrument is steadily increased from a negative value, the output reading varies in the manner shown in curve (A). If the input variable is then steadily decreased, the output varies in the manner shown in curve (B). The noncoincidence between these loading and unloading curves is known as hysteresis. Two quantities are defined, maximum input hysteresis and maximum output hysteresis, as shown in Figure 2.4. These are normally expressed as a percentage of the full scale input or output reading, respectively.

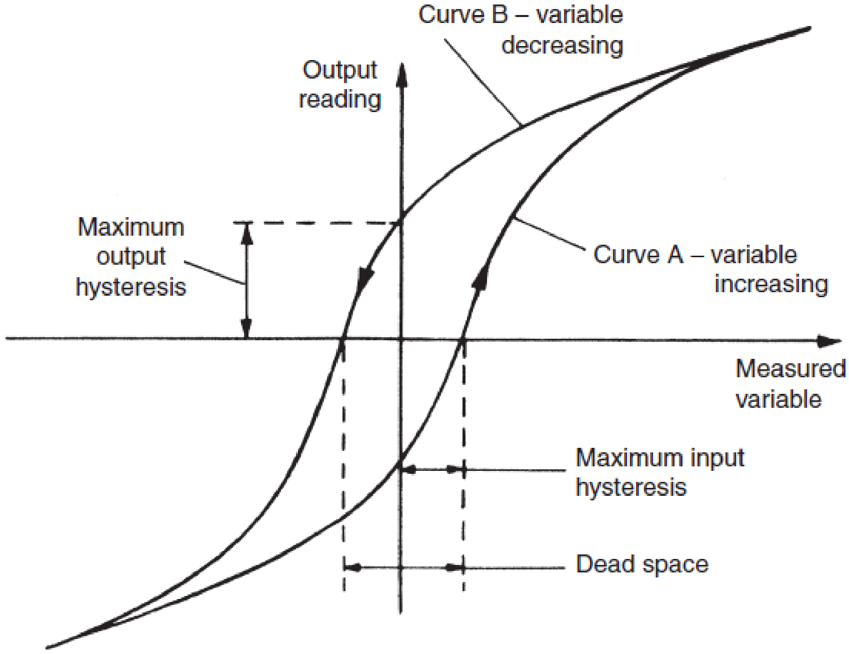

Figure 2.4 Instrument characteristic with hysteresis.

2.2.11 Dead Space

Dead space is defined as the range of different input values over which there is no change in output value. Any instrument that exhibits hysteresis also displays dead space, as marked on Figure 2.4.

2.3 Dynamic Characteristics of Instruments

The static characteristics of measuring instruments are concerned only with the steady state reading that the instrument settles down to, such as the accuracy of the reading, etc.

The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response.

As with static characteristics, any values for dynamic characteristics quoted in instrument data sheets only apply when the instrument is used under specified environmental conditions. Outside these calibration conditions, some variation in the dynamic parameters can be expected.

In any linear, time-invariant measuring system, the following general relation can be written between input and output for time (t) > 0:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_m \frac{d^m q_i}{dt^m} + b_{m-1} \frac{d^{m-1} q_i}{dt^{m-1}} + \dots + b_1 \frac{dq_i}{dt} + b_0 q_i$$

Where **qi** is the measured quantity, **qo** is the output reading, and a0.an, b0.bm are constants.

If we limit consideration to that of step changes in the measured quantity only, then Eqn (2.1) reduces to:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \dots + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

Further simplification can be made by taking certain special cases of Eqn (2.1), which collectively apply to nearly all measurement systems.

2.3.1 Zero-Order Instrument

If all the coefficients $a_1 \dots a_n$ other than a_0 in Eqn (2.2) are assumed zero, then:

$$a_0 q_o = b_0 q_i \quad or \quad q_o = b_0 q_i / a_0 = K q_i \tag{2.3}$$

where K is a constant known as the instrument sensitivity as defined earlier.

Any instrument that behaves according to Eqn (2.3) is said to be of zero-order type.

Following a step change in the measured quantity at time t, the instrument output moves immediately to a new value at the same time instant t, as shown in Figure 2.10. A potentiometer, which measures motion, is a good example of such an instrument, where the output voltage changes instantaneously as the slider is displaced along the potentiometer track.

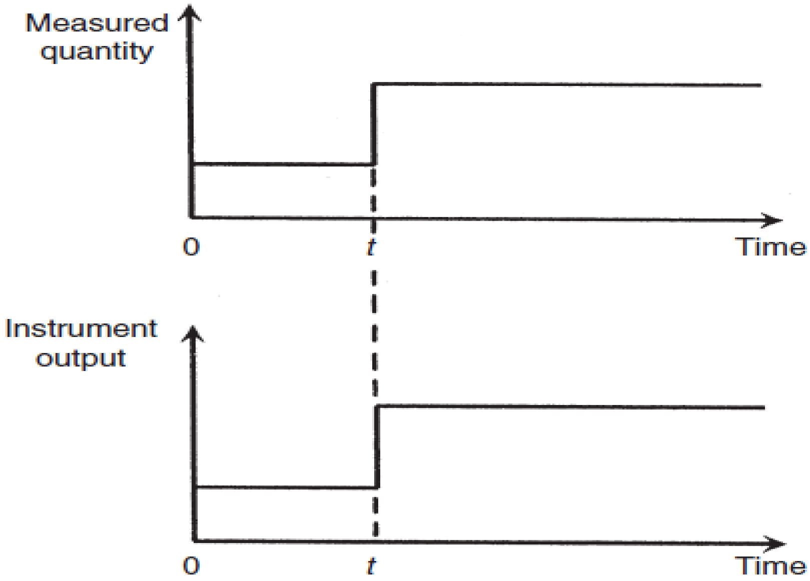

Figure 2.10
Zero-order instrument characteristic.

2.3.2 First-Order Instrument

If all the coefficients a2…an except for a0 and a1 are assumed zero in Eqn (2.2) then:

$$a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \qquad (2.4)$$

Any instrument that behaves according to Eqn (2.4) is known as a first-order instrument. If d/dt is replaced by the D operator in Eqn (2.4), we get:

$$a_1 D q_o + a_0 q_o = b_0 q_i \quad \text{and rearranging this gives:} \quad q_o = \frac{(b_0/a_0) q_i}{[1 + (a_1/a_0) D]} \tag{2.5}$$

Defining $K = b_0/a_0$ as the static sensitivity and $\tau = a_1/a_0$ as the time constant of the system, Eqn (2.5) becomes:

$$q_o = \frac{K q_i}{1 + \tau D} \tag{2.6}$$

If Eqn (2.6) is solved analytically, the output quantity qo in response to a step change in qi at time t varies with time in the manner as shown in Figure 2.11. The time constant s of the step response is the time taken for the output quantity qo to reach 63% of its final value.

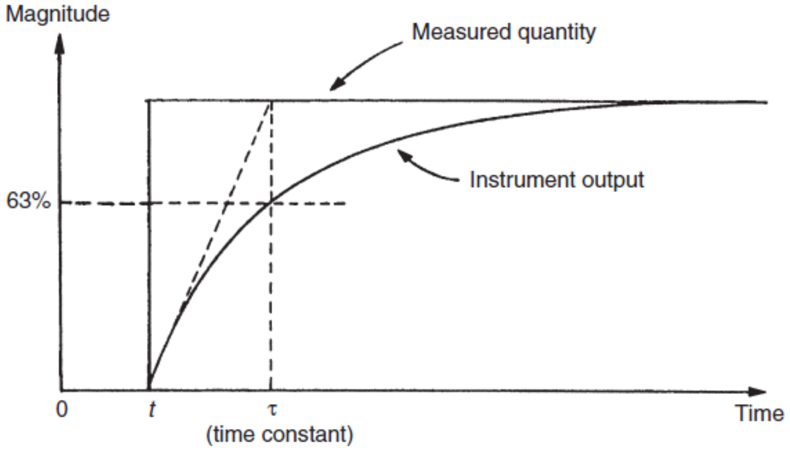

Figure 2.11
First-order instrument characteristic.

2.3.3 Second –order instrument

If all coefficients a3..an other than a0, a1, and a2 in Eqn (2.2) are assumed zero, then we get:

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i \tag{2.7}$$

Applying the D operator again: $a_2 D^2 q_o + a_1 D q_o + a_0 q_o = b_0 q_i$, and rearranging:

$$q_o = \frac{b_0 q_i}{a_0 + a_1 D + a_2 D^2} \tag{2.8}$$

It is convenient to reexpress the variables a_0, a_1, a_2, and b_0 in Eqn (2.8) in terms of three parameters K (static sensitivity), ω (undamped natural frequency), and ξ (damping ratio), where:

$$K = b_0/a_0 \quad ; \quad \omega = \sqrt{a_0/a_2} \quad ; \quad \xi = a_1/2\sqrt{a_0 a_2}$$

ξ can be written as $\xi = \frac{a_1}{2a_0\sqrt{a_2/a_0}} = \frac{a_1\omega}{2a_0}$.

If Eqn (2.8) is now divided through by a_0 we get:

$$q_o = \frac{(b_0/a_0)q_i}{1 + (a_1/a_0)D + (a_2/a_0)D^2} \tag{2.9}$$

The terms in Eqn (2.9) can be written in terms of ω and ξ as follows:

$$\frac{b_0}{a_0} = K \quad ; \quad \left(\frac{a_1}{a_0}\right)D = \frac{2\xi D}{\omega} \quad ; \quad \left(\frac{a_2}{a_0}\right)D^2 = \frac{D^2}{\omega^2}$$

Hence, dividing Eqn (2.9) through by q_i and substituting for a_0, a_1, and a_2 gives:

$$\frac{q_0}{q_i} = \frac{K}{D^2/\omega^2 + 2\xi D/\omega + 1} \tag{2.10}$$

This is the standard equation for a second-order system and any instrument whose response can be described by it is known as a second-order instrument. If Eqn (2.10) is solved analytically, the shape of the step response obtained depends on the value of the damping ratio parameter ξ. The output responses of a second-order instrument for various values of ξ following a step change in the value of the measured quantity at time t are shown in Figure 2.12.

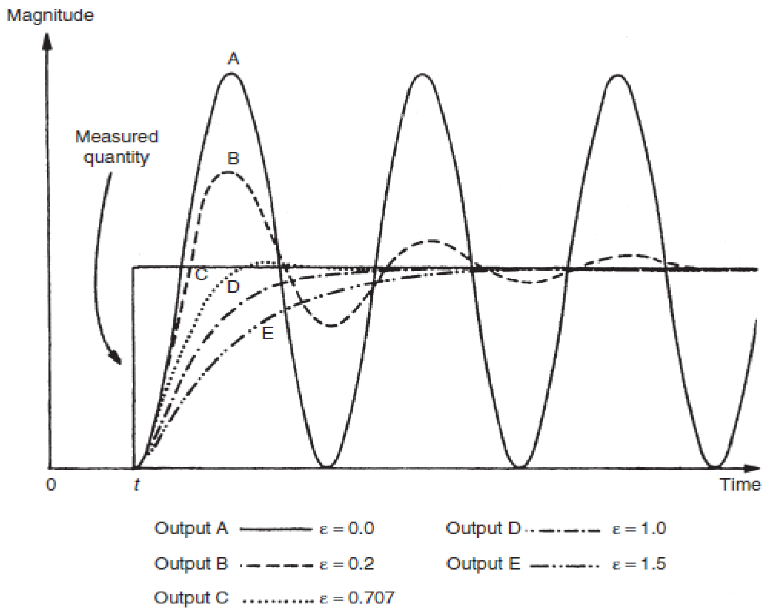

Figure 2.12
Response characteristics of second-order instruments.