Comparison of Amplitude Modulation Techniques

	Modulator	**Demodulator**	**Power Efficiency**	**Bandwidth**
DSB-LC (audio broadcasting)	Simple	Simple (envelope)	<1/3 (single tone)	2B
DSB-SC	Simple	Complex (coherent)	1	2B
SSB-SC (telephony- wirelines and cables)	Complex	Complex (coherent)	1	B
SSB-LC (TV transmission)	Complex	Simple (envelope)	Worst (very large carrier)	B

Example 1: A DSB-LC signal has the form

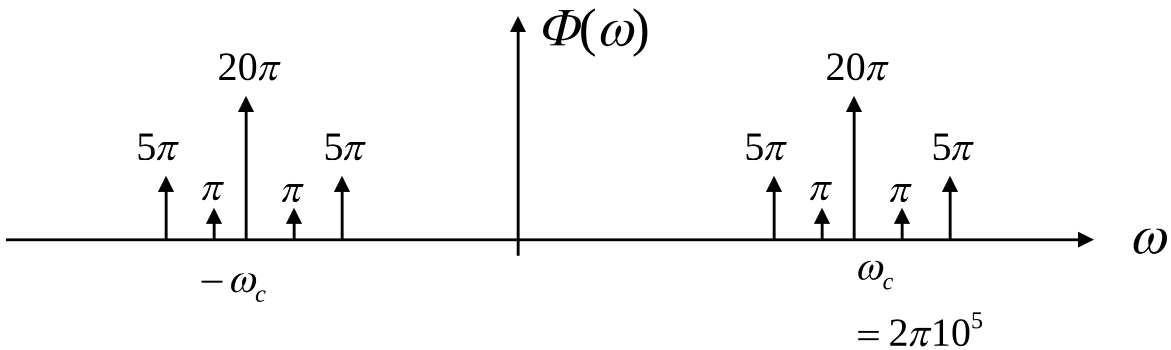
$$\phi(t) = [20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)]\cos(2\pi f_c t) \qquad f_c = 10^5 \text{ Hz}$$

a) Determine the Fourier transform of $\phi(t)$

$$\phi(t) = 20\cos(2\pi f_c t) + \cos(2\pi f_c t - 3000\pi t) + \cos(2\pi f_c t + 3000\pi t)$$
$$+ 5\cos(2\pi f_c t - 6000\pi t) + 5\cos(2\pi f_c t + 6000\pi t)$$

$$\Phi(\omega) = \Im[\phi(t)]$$

$$= 20\pi[\delta(\omega-\omega_c) + \delta(\omega-\omega_c)] + \pi[\delta(\omega-\omega_c+3000\pi) + \delta(\omega+\omega_c-3000\pi)]$$
$$+ \pi[\delta(\omega-\omega_c-3000\pi) + \delta(\omega+\omega_c+3000\pi)] + 5\pi[\delta(\omega-\omega_c+6000\pi) + \delta(\omega+\omega_c-6000\pi)]$$
$$+ 5\pi[\delta(\omega-\omega_c-6000\pi) + \delta(\omega+\omega_c+6000\pi)]$$



b) Determine the power in each of the frequency components.

$$\phi(t) = 20\cos(2\pi f_c t) + \cos(2\pi f_c t - 3000\pi t) + \cos(2\pi f_c t + 3000\pi t)$$
$$+ 5\cos(2\pi f_c t - 6000\pi t) + 5\cos(2\pi f_c t + 6000\pi t)$$

$P_1 = 20^2/2 = 200$ $\quad$ $P_4 = (5)^2/2 = 12.5$

$P_2 = (1)^2/2 = 0.5$ $\quad$ $P_5 = (5)^2/2 = 12.5$

$P_3 = (1)^2/2 = 0.5$

c) Determine the modulation index

$$\phi(t) = A_c(1 + f(t)/A_c)\cos(\omega_c t)$$

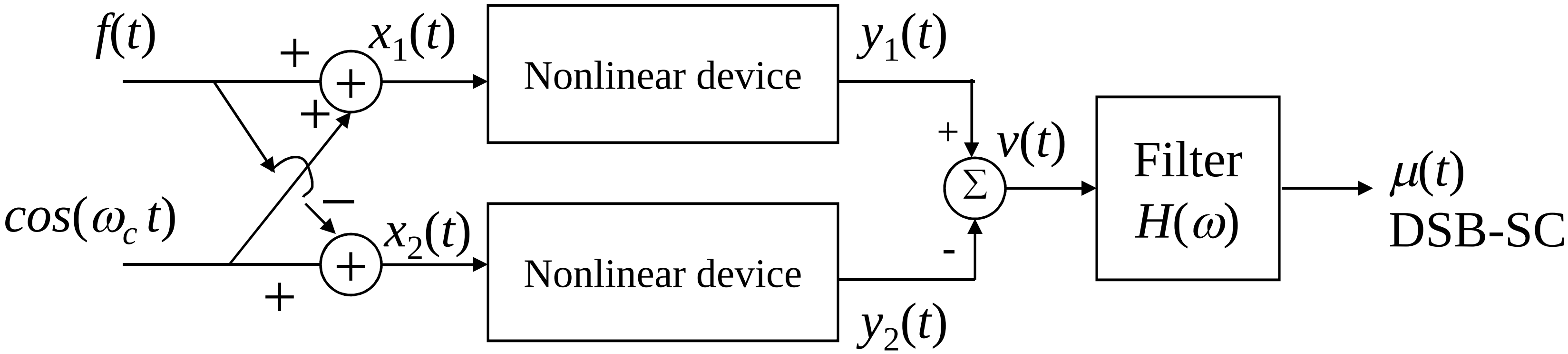
$$f(t) = 2\cos(3000\pi t) + 10\cos(6000\pi t)$$

$$m = \frac{\max|f(t)|}{A_c} = \frac{12}{20} = 0.6$$

d) Determine the modulation efficiency

$$\mu = \frac{\text{useful power}}{\text{total power}} = \frac{P_2 + P_3 + P_4 + P_5}{P_1 + P_2 + P_3 + P_4 + P_5} = \frac{26}{226} = 0.115$$

Example 2



Non-linear devices are described by the following input-output relations

$$y_1(t) = ax_1(t) + bx_1^2(t)$$
$$y_2(t) = ax_2(t) + bx_2^2(t)$$

a) What is the input signal of the filter?

$$\left.\begin{aligned} x_1(t) &= f(t) + \cos(\omega_c t) \\ x_2(t) &= -f(t) + \cos(\omega_c t) \end{aligned}\right\} \quad \begin{aligned} v(t) &= y_1(t) - y_2(t) \\ &= 2af(t) + 4bf(t)\cos(\omega_c t) \end{aligned}$$

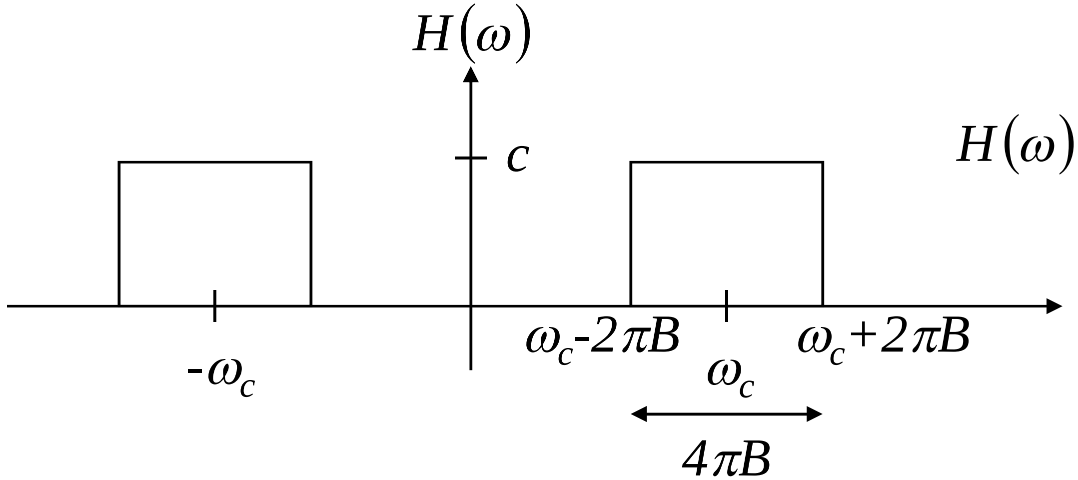
$$V(\omega) = 2aF(\omega) + 2b[F(\omega - \omega_c) + F(\omega + \omega_c)]$$

b) Specify the transfer function of the filter.

In order to obtain the DSB-SC signal, we need a band-pass filter which will only allow the second term.

$$v(t) = 2af(t) + 4bf(t)\cos(\omega_c t)$$

$$V(\omega) = 2aF(\omega) + 2b[F(\omega - \omega_c) + F(\omega + \omega_c)]$$



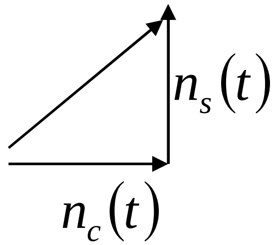
$$H(\omega) = \begin{cases} c, & |\omega - \omega_c| \le 2\pi B \\ 0, & \text{otherwise} \end{cases}$$

c) What is the output signal of the filter?

$$\mu(t) = 4bcf(t)\cos(\omega_c t)$$

Time Representations for Bandpass Noise

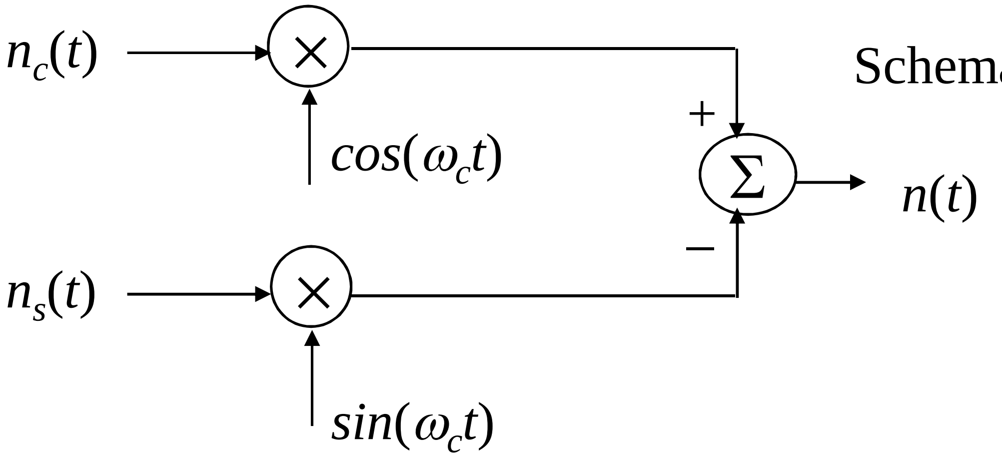
$$n(t) = \mathrm{Re}\left\{[n_c(t) + jn_s(t)]e^{j\omega_c t}\right\} = n_c(t)\cos(\omega_c t) - n_s(t)\sin(\omega_c t)$$



n_c : In-phase noise component

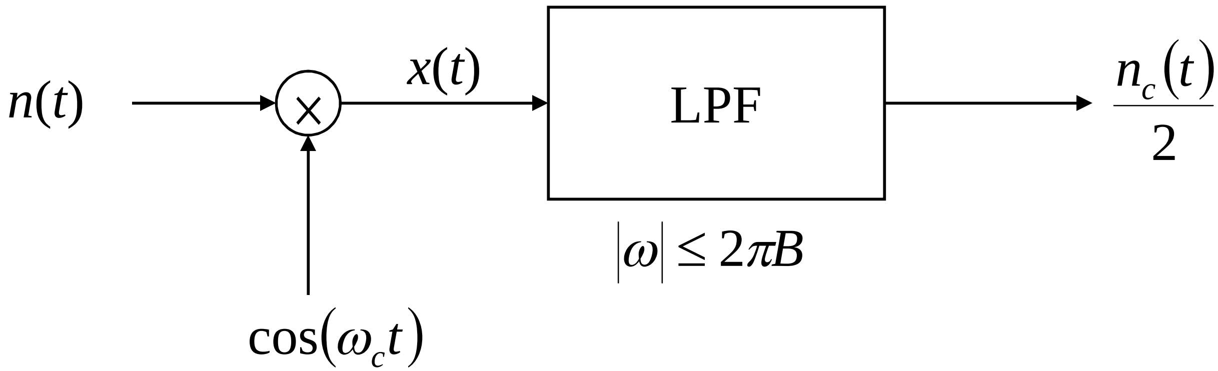
n_s : Quadrature noise component

} Low-pass noise components



Schematic representation of noise

How to obtain low-pass noise components

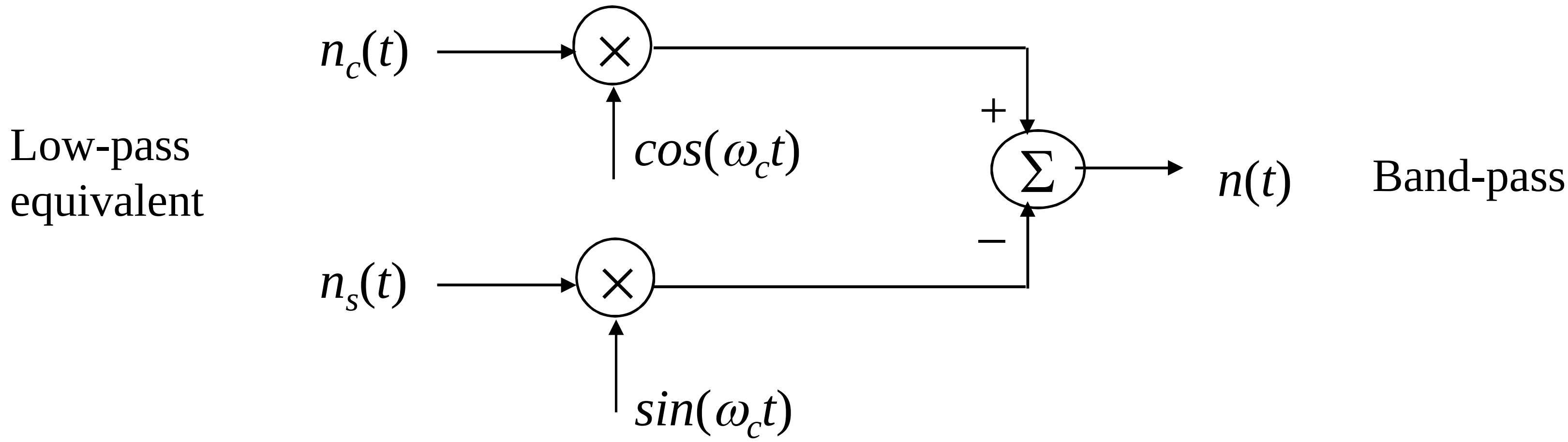


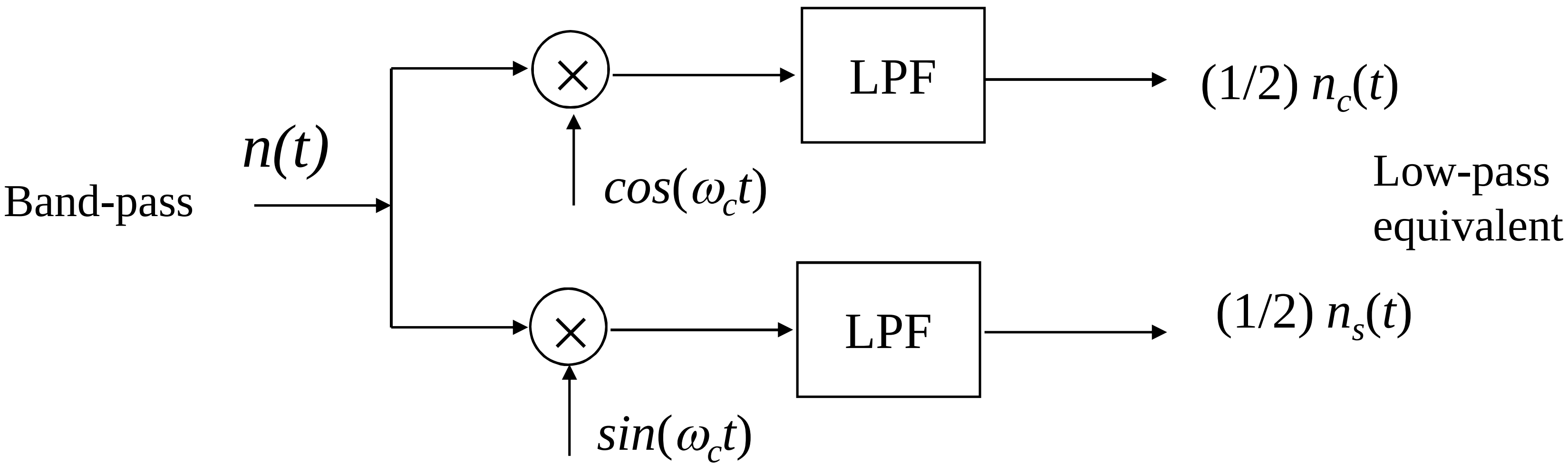
$$
\begin{aligned}
x(t) &= n(t)\cos(\omega_c t) \\
&= n_c(t)\cos^2(\omega_c t) - n_s(t)\cos(\omega_c t)\sin(\omega_c t) \\
&= \underbrace{\frac{n_c(t)}{2}} + \frac{n_c(t)}{2}\cos(2\omega_c t) - \frac{n_s(t)}{2}\sin(2\omega_c t)
\end{aligned}
$$

$$
\sin\alpha \cdot \cos\beta = (1/2)[\sin(\alpha-\beta) + \sin(\alpha+\beta)]
$$

Only this term passes through LPF

Schematic representations of noise

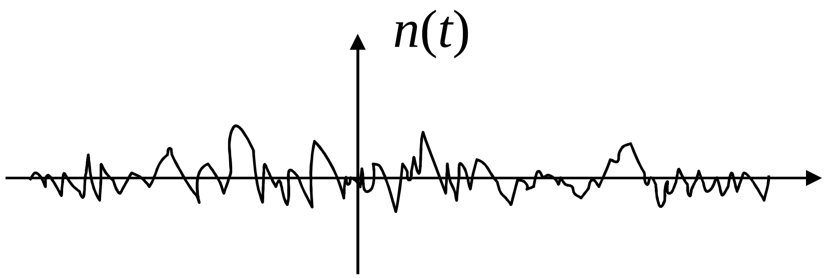


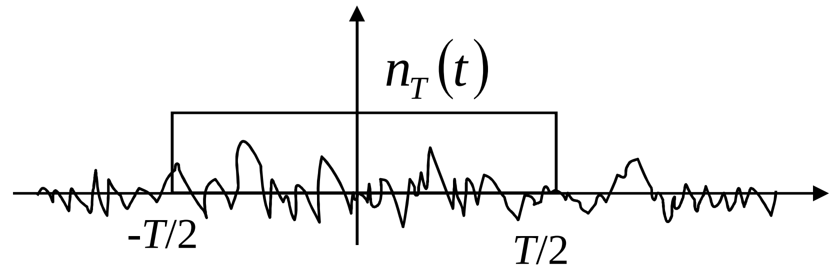


Spectral Density of $n_c(t)$ and $n_s(t)$

Recall the definition for power spectral density

$$S_n(\omega) = \lim_{T\to\infty} \frac{|N_T(\omega)|^2}{T}$$





$$n_T(t) = n(t)\,\mathrm{rect}(t/T)$$

$$n_c(t) = 2[n(t)\cos(\omega_c t)]_{LPF}$$

$$S_{n_c}(\omega) = \left\{ \lim_{T\to\infty} \frac{|N_T(\omega-\omega_c) + N_T(\omega+\omega_c)|^2}{T} \right\}_{LPF}$$

For random noise, cross-product terms are equal to zero.

$$= \left\{ \lim_{T\to\infty} \frac{|N_T(\omega-\omega_c)|^2}{T} + \frac{|N_T(\omega+\omega_c)|^2}{T} \right\}_{LPF} = \{S_n(\omega-\omega_c) + S_n(\omega+\omega_c)\}_{LPF}$$

Spectral Density of $n_c(t)$ **and** $n_s(t)$ (Cont'd)

$$S_{n_C}(\omega) = S_{n_S}(\omega) = \{S_n(\omega - \omega_c) + S_n(\omega + \omega_c)\}_{LPF}$$

$$= \begin{cases} S_n(\omega - \omega_c) + S_n(\omega + \omega_c), & |\omega| \le 2\pi B \\ 0, & \text{otherwise} \end{cases}$$

where

$$S_n(\omega) \ne 0, \text{ for } |\omega| \in [\omega_c - 2\pi B,\ \omega_c + 2\pi B]$$

The area under $S_n(\omega)$ is equal to the area under $S_{n_C}(\omega)$ and $S_{n_s}(\omega)$