Representations for SSB Signals

$$\phi_{\mathrm{SSB}+} = 2\,\mathrm{Re}\left\{ f_{+}(t) e^{j\omega_c t} \right\}$$

$$\phi_{\mathrm{SSB}-} = 2\,\mathrm{Re}\left\{ f_{-}(t) e^{j\omega_c t} \right\}$$

\+ Upper sideband

\- Lower sideband

$$\phi_{\mathrm{SSB}+} = \mathrm{Re}\left\{ \left[f(t) + j\hat{f}(t) \right] e^{j\omega_c t} \right\}$$

$$\phi_{\mathrm{SSB}-} = \mathrm{Re}\left\{ \left[f(t) - j\hat{f}(t) \right] e^{j\omega_c t} \right\}$$

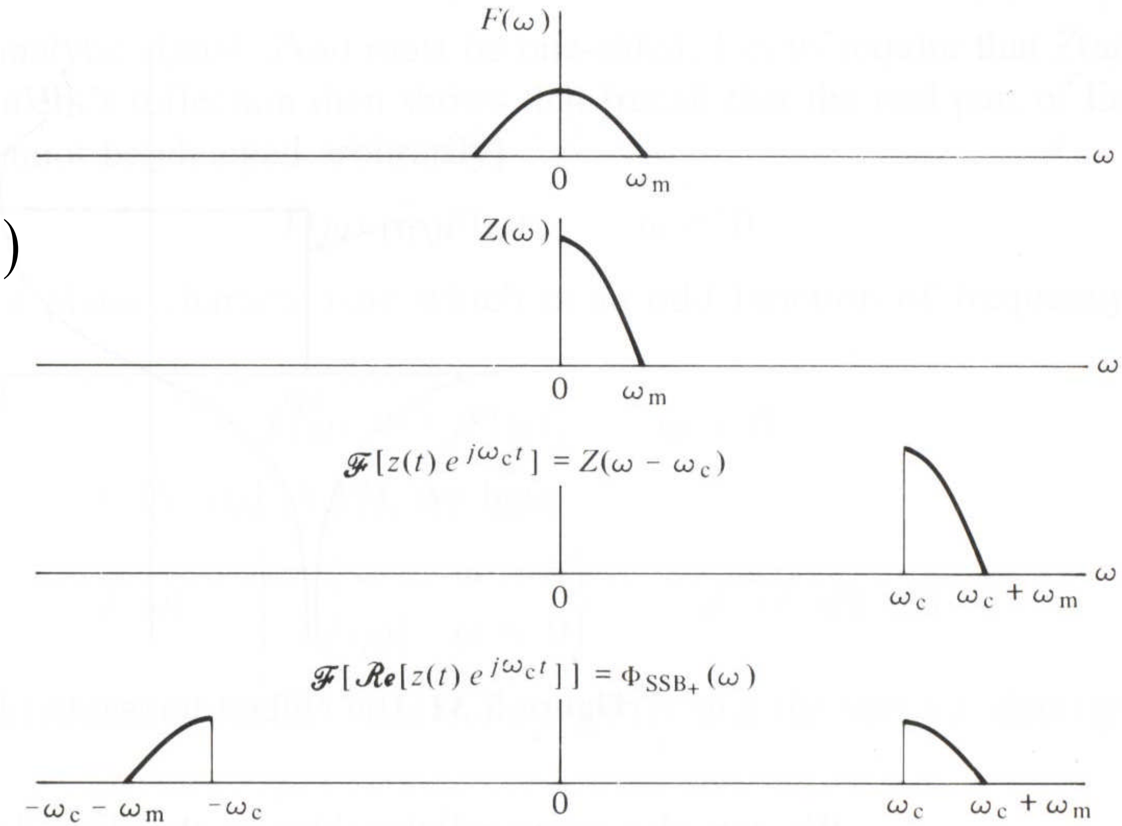
$$\phi_{\mathrm{SSB}+} = f(t)\cos(\omega_c t) - \hat{f}(t)\sin(\omega_c t)$$

$$\phi_{\mathrm{SSB}-} = f(t)\cos(\omega_c t) + \hat{f}(t)\sin(\omega_c t)$$

SSB (Upper Sideband)

$$z(t) = f(t) + j\hat{f}(t)$$

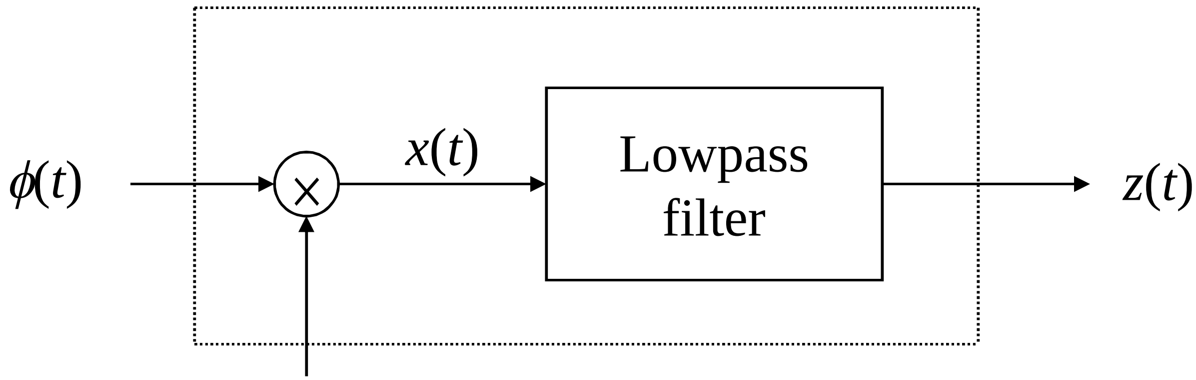
$$z(t) \longleftrightarrow Z(\omega)$$



$$\phi_{\text{SSB+}}(t) = \text{Re}\left\{ z(t)\; e^{j\omega_c t} \right\} = f(t)\cos(\omega_c t) - \hat{f}(t)\sin(\omega_c t)$$

Demodulation of SSB signals

Given $\phi(t)$, how will be the message signal $f(t)$ be recovered?



$\cos(\omega_c t)$: Locally generated carrier signal

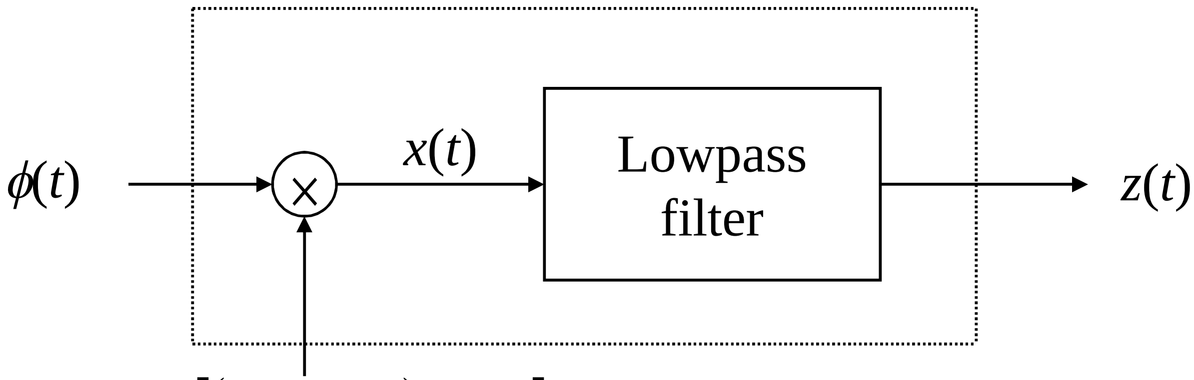
$$x(t) = \phi_{\pm}(t)\cos(\omega_c t)$$

$$= \left[f(t)\cos(\omega_c t) \mp \hat{f}(t)\sin(\omega_c t) \right] \cos(\omega_c t)$$

$$= \underbrace{\frac{1}{2} f(t)} + \frac{1}{2} f(t)\cos(2\omega_c t) \mp \frac{1}{2}\hat{f}(t)\sin(2\omega_c t)$$

Only this term passes
through LPF

Now assume a frequency error and a phase error in the locally generated signal at the receiver.



$\cos\left[(\omega_c + \Delta\omega)t + \theta_0\right]$: Locally generated carrier signal

$$x(t) = \left[f(t)\cos(\omega_c t) \mp \hat{f}(t)\sin(\omega_c t)\right]\cos\left[(\omega_c + \Delta\omega)t + \theta_0\right]$$

$$= \frac{1}{2} f(t)\left\{\cos\left[(\Delta\omega)t + \theta_0\right] + \cos\left[(2\omega_c + \Delta\omega)t + \theta_0\right]\right\}$$

$$\underbrace{\pm \frac{1}{2} \hat{f}(t)\left\{\sin\left[(\Delta\omega)t + \theta_0\right]\right.}_{\text{Only these two terms pass through LPF}} \left. - \sin\left[(2\omega_c + \Delta\omega)t + \theta_0\right]\right\}$$

$\cos\alpha \cdot \cos\beta = (1/2)\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$

$\sin\alpha \cdot \cos\beta = (1/2)\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$

$$z(t)=\frac{1}{2}f(t)\cos[(\Delta\omega)t+\theta_0]\pm\frac{1}{2}\hat{f}(t)\sin[(\Delta\omega)t+\theta_0]$$

- If $\Delta\omega$=0 and θ_0= 0, the output is $z(t)=\frac{1}{2}f(t)$ → no distortion

- If $\Delta\omega$ =0, the output is $z(t)=\frac{1}{2}f(t)\cos(\theta_0)\pm\frac{1}{2}\hat{f}(t)\sin(\theta_0)$
$$=\frac{1}{2}\mathrm{Re}\left\{\left[f(t)\pm\hat{f}(t)\right]e^{j\theta}\right\}$$

Human ear can interpret speech despite phase changes.

- If θ_0= 0, the output is $z(t)=\frac{1}{2}f(t)\cos[(\Delta\omega)t]\pm\frac{1}{2}\hat{f}(t)\sin[(\Delta\omega)t]$
$$=\frac{1}{2}\mathrm{Re}\left\{\left[f(t)\pm\hat{f}(t)\right]e^{j(\Delta\omega)t}\right\}$$

Frequency errors result in spectral shifts. Small errors can be tolerated in voice communication.

Single Sideband Large Carrier (SSB-LC)

Coherent (synchronous) detection is required for SSB. Inserting a carrier, envelope detection can be used.

SSB $\quad \phi_{\pm}(t) = f(t)\cos(\omega_c t) \pm \hat{f}(t)\sin(\omega_c t)$

SSB-LC $\quad \phi(t) = A_c \cos(\omega_c t) + f(t)\cos(\omega_c t) \pm \hat{f}(t)\sin(\omega_c t)$

$$= [A_c + f(t)]\cos(\omega_c t) \pm \hat{f}(t)\sin(\omega_c t)$$

Envelope of $\phi(t)$

$$r(t) = \sqrt{[A_c + f(t)]^2 + \left[\hat{f}(t)\right]^2} = \sqrt{A_c^2 + 2f(t)A_c + f^2(t) + \hat{f}^2(t)}$$

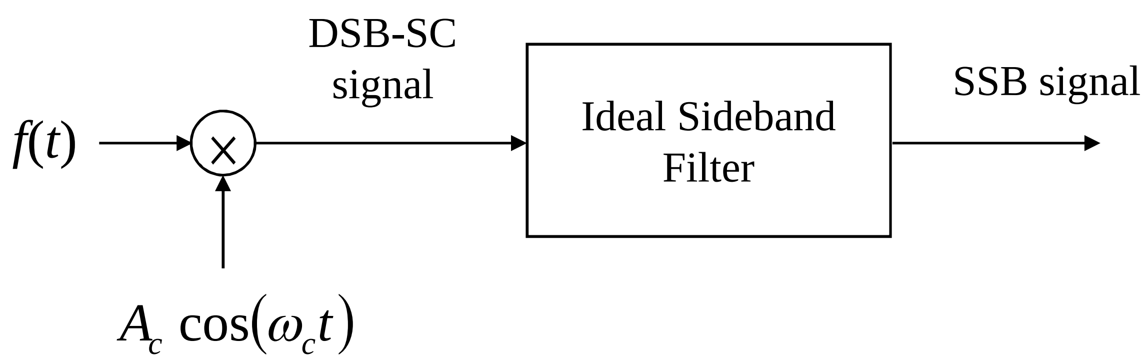
$$\approx A_c\sqrt{1 + \frac{2f(t)}{A_c}} \approx A_c\left[1 + \frac{f(t)}{A_c}\right] = A_c + f(t)$$

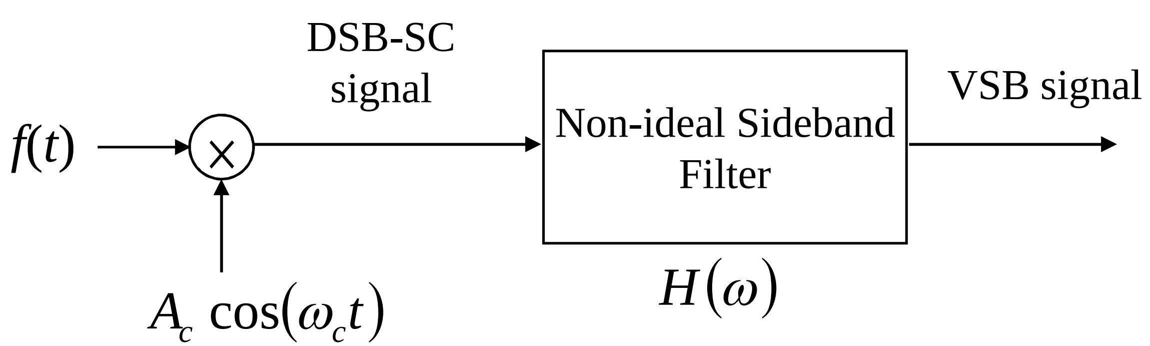
For very large A_c $\qquad \sqrt{1+x} \approx 1 + x/2 \quad |x| << 1$

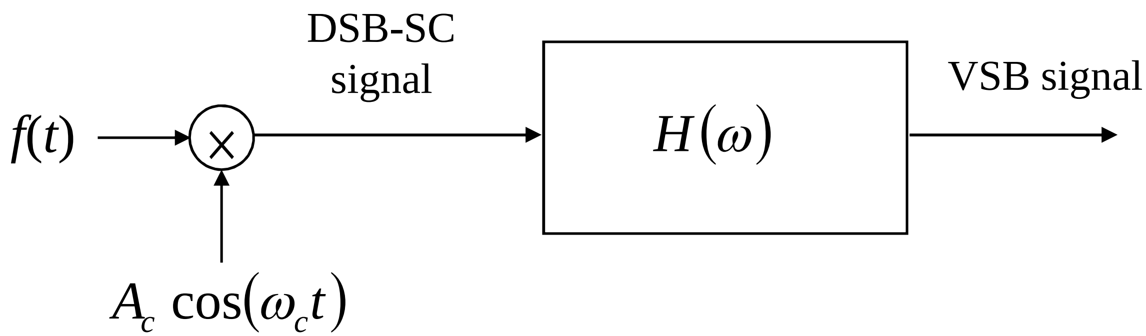
Vestigial Sideband (VSB) Modulation

(5.5 in Textbook)

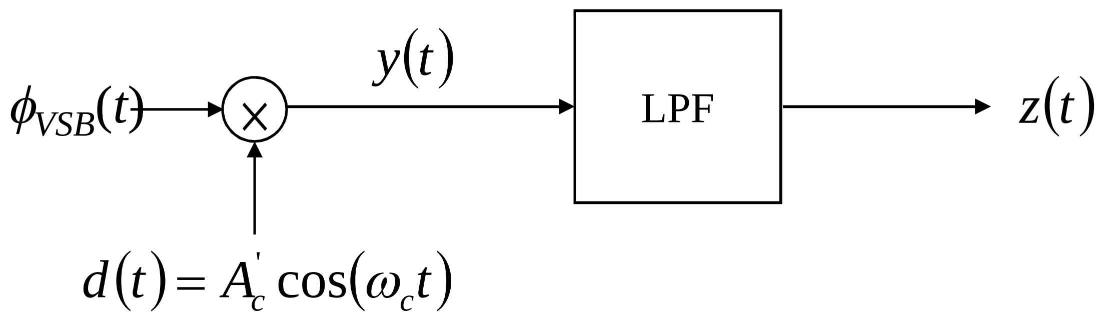
As a compromise between bandwidth efficiency and hardware cost (between DSB and SSB), we can use VSB modulation. In this modulation type, one sideband is transmitted, also allowing a portion of the unwanted sideband.







To determine the frequency-response characteristics of the filter, let us consider the demodulation for VSB signal.



$$\phi_{VSB}(t) = [A_c f(t)\cos\omega_c t] \otimes h(t) \qquad \Phi_{VSB}(\omega) = \frac{A_c}{2}[F(\omega-\omega_c) + F(\omega+\omega_c)]H(\omega)$$

$$y(t) = \phi_{VSB}(t) d(t) \qquad Y = \frac{A_c'}{2}[\Phi_{VSB}(\omega-\omega_c) + \Phi_{VSB}(\omega+\omega_c)]$$

$$Y(\omega)=\frac{A_c'}{2}\left[\Phi_{VSB}(\omega-\omega_c)+\Phi_{VSB}(\omega+\omega_c)\right]$$

$$=\frac{A_cA_c'}{4}F(\omega)\left[H(\omega-\omega_c)+H(\omega+\omega_c)\right]+\cancelto{0}{\frac{A_cA_c'}{4}F(\omega-2\omega_c)H(\omega-\omega_c)}$$

$$+\cancelto{0}{\frac{A_cA_c'}{4}F(\omega+2\omega_c)H(\omega+\omega_c)}$$

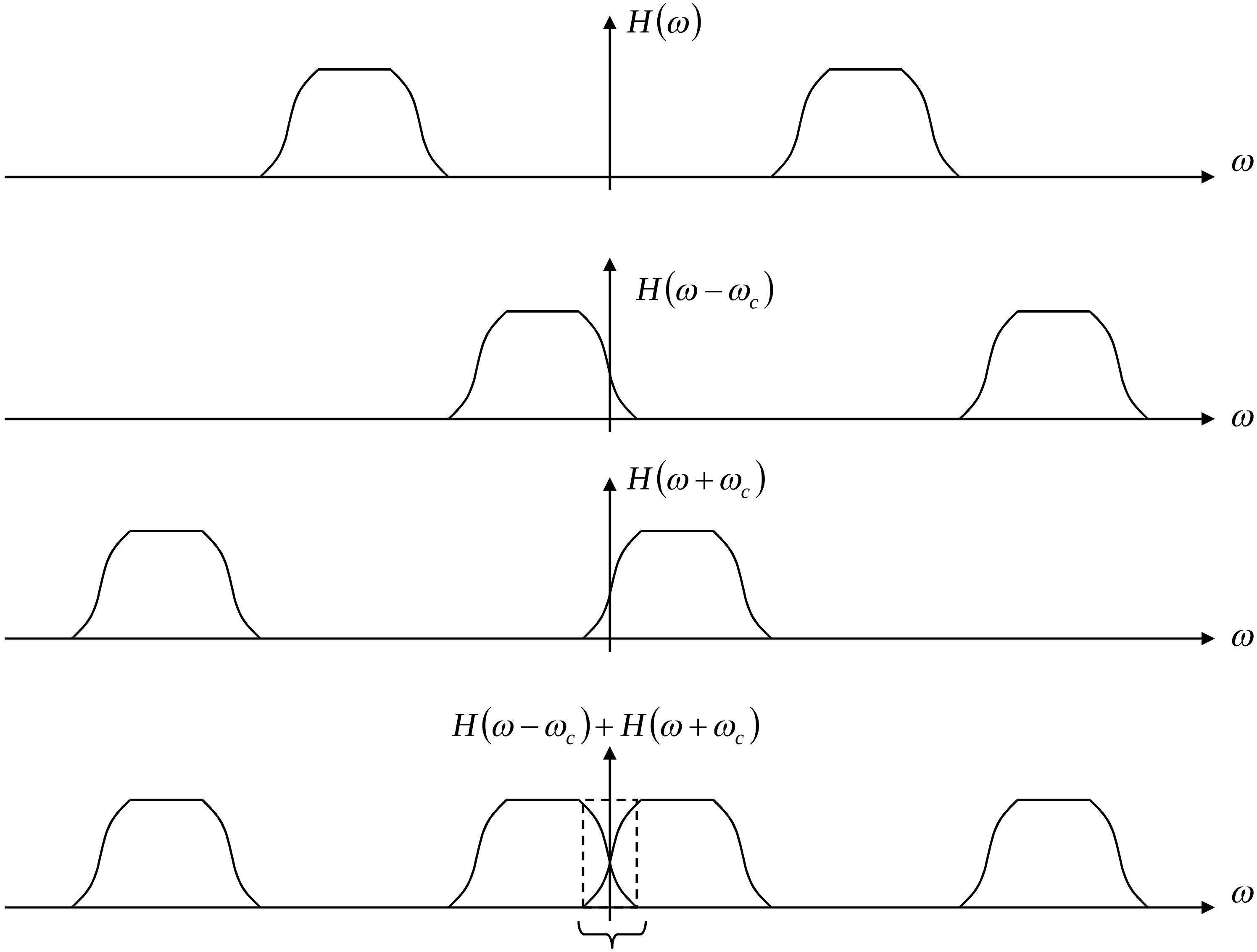
Output of LPF

$$Z(\omega)=\frac{A_cA_c'}{4}F(\omega)\left[H(\omega-\omega_c)+H(\omega+\omega_c)\right]$$

We require that the message signal at the output of LPF be undistorted.

$$H(\omega-\omega_c)+H(\omega+\omega_c)=\text{constant} \qquad |\omega|\leq \text{Bandwidth of the message signal}$$

Example for VSB Filter



constant $\rightarrow$ Satisfies the requirement for distortionless demodulation