Useful components in $V_2(t)$ are $\frac{1}{2}A_c\cos(\omega_c t)$ and $\frac{2}{\pi}f(t)\cos(\omega_c t)$

Use a bandpass filter centered at ω_c with bandwidth $2W$ ($W=2\pi B$) to extract the useful components.

$$V_2(t)=\frac{1}{2}f(t)+\frac{1}{2}A_c\cos(\omega_c t)+\frac{2}{\pi}\cos^2(\omega_c t)+\frac{2}{\pi}f(t)\cos(\omega_c t)$$
$$-\frac{2}{3\pi}A_c\cos(\omega_c t)\cos(3\omega_c t)-\frac{2}{3\pi}f(t)\cos(3\omega_c t)+\text{other terms}$$

Only these terms go through the bandpass filters

Output of BPF: $$V_0(t)=\frac{1}{2}A_c\cos(\omega_c t)+\frac{2}{\pi}f(t)\cos(\omega_c t)$$

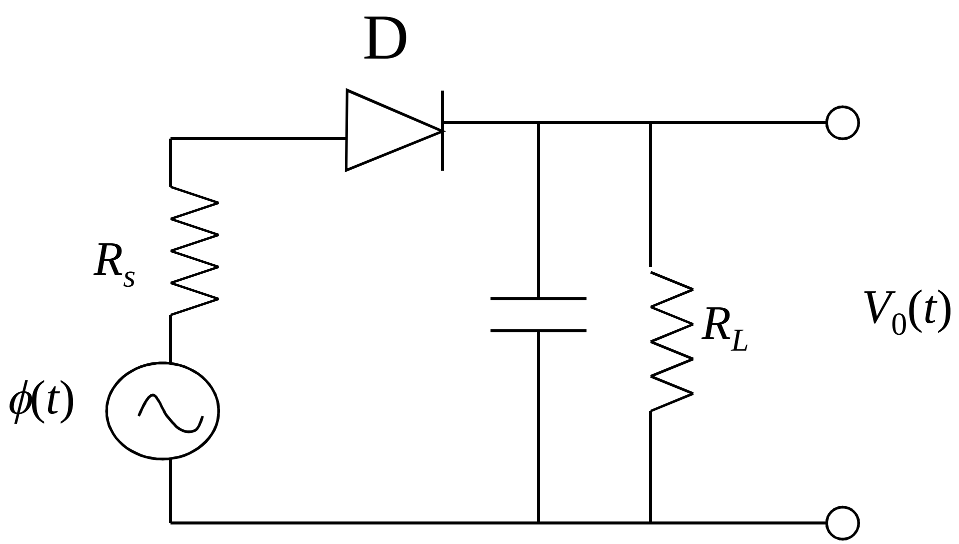
$V_0(t)$ is DSB-LC modulated signal $\phi(t)$ with some scaling factors.

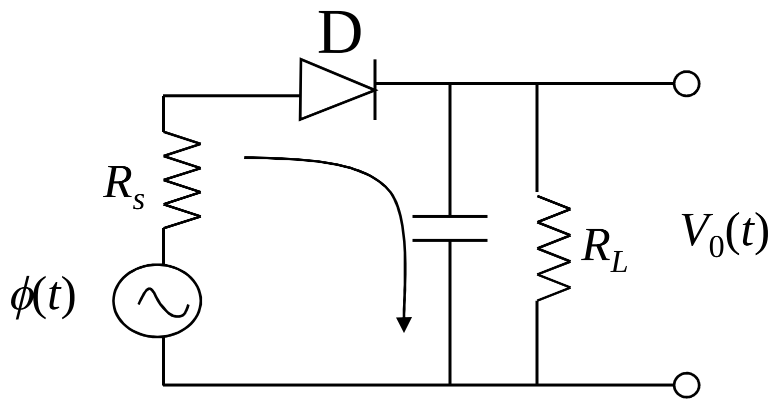
Demodulation of DSB-LC Signals

Circuitry aspects of this topic will not be in any examination, mathematical aspects may be tested.

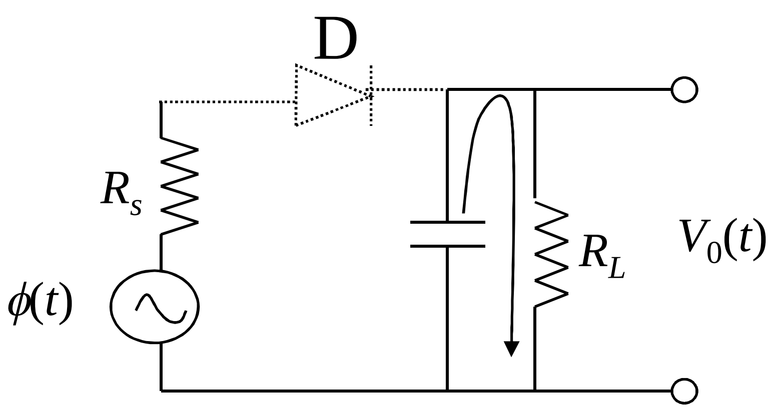
Given $\phi(t)$, how will be the message signal $f(t)$ be recovered?

Envelope Detector





$\phi(t) > V_0(t)$
The diode is forward biased
C charges up (time constant $\tau_1 = R_s C$)
Follows the variation of input signal $\phi(t)$
$\tau_1 << 1/f_c$ $(\omega_c = 2\pi f_c)$

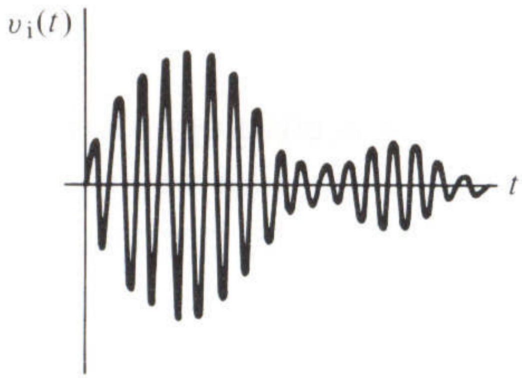


$\phi(t) < V_0(t)$
The diode turns off
C discharges (time constant $\tau_2 = R_L C$)

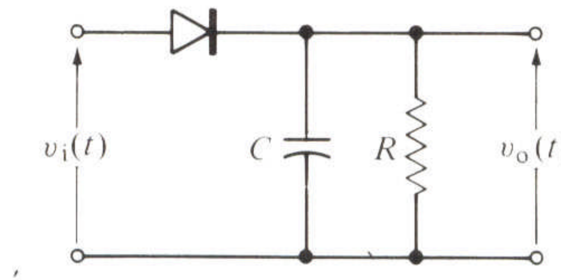
Choice of $R_L C$

$$\frac{1}{f_c} << \tau_2 << \frac{1}{B}$$

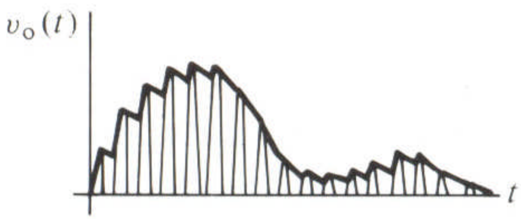
B is the bandwidth of $f(t)$.



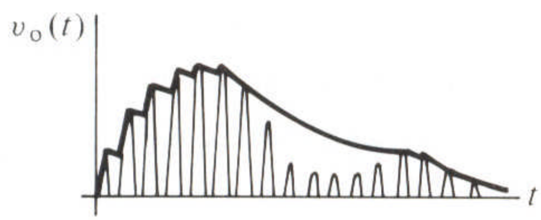
(a)



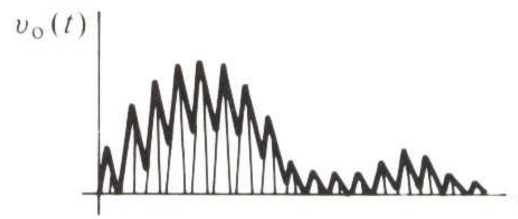
(b)



(c) Correct *RC*



(d) *RC* too large



(e) *RC* too small

$R_L C$ too large: Envelope detector misses some positive half cycles.

$R_L C$ too small: Envelope generates a very ragged waveform.

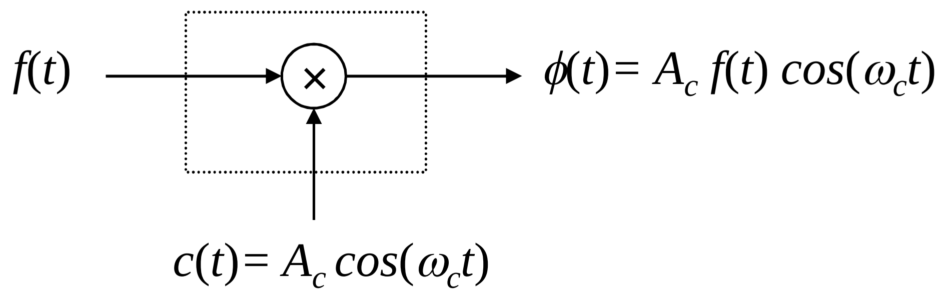
Comments on DSB-LC

Both the modulator and demodulator have simple structures (low cost).
DSB-LC is wasteful of power (carrier does not carry information).
DSB-LC is wasteful of bandwidth (*B* vs 2*B*)

To overcome the drawbacks, we have

- Double Sideband Suppressed Carrier (DSB-SC) Modulation
- Single Sideband (SSB) Modulation
- Vestigial Sideband (VSB) Modulation

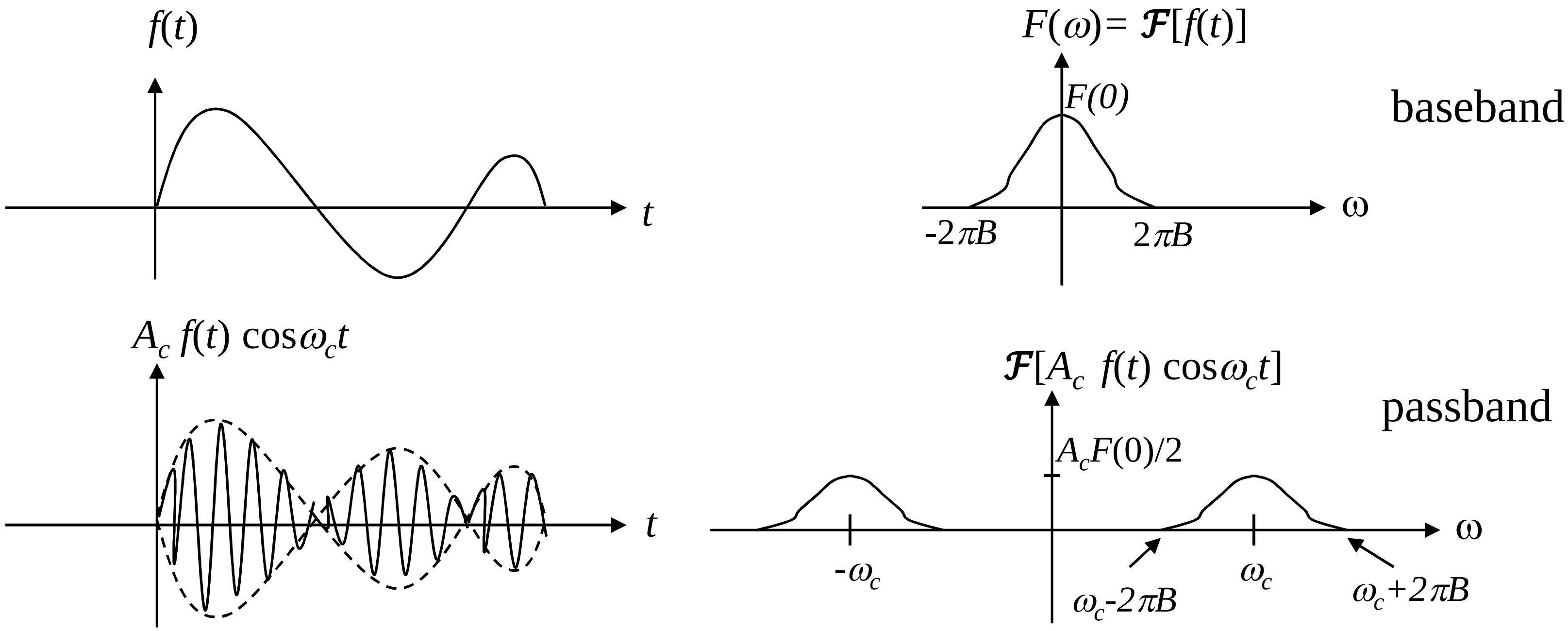
Double Side Band Supressed Carrier (DSB-SC) (5.1 in Textbook)



$$F(\omega) = \mathcal{F}\{f(t)\} \qquad \Phi(\omega) = \mathcal{F}\{\phi(t)\}$$

$$\begin{aligned}\Phi(\omega) &= \mathcal{F}\left[A_c f(t)\cos(\omega_c t)\right] \\ &= \mathcal{F}\left(\frac{A_c}{2} f(t) e^{j\omega_c t} + \frac{A_c}{2} f(t) e^{-j\omega_c t}\right) \\ &= \frac{A_c}{2} F(\omega - \omega_c) + \frac{A_c}{2} F(\omega + \omega_c)\end{aligned}$$

$$\cos\omega_c t = \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$$



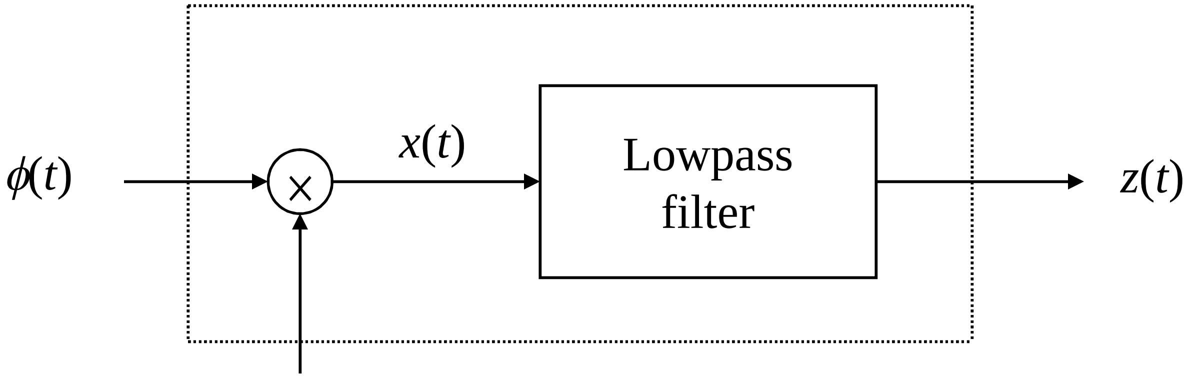
Observations:

- $\phi(t)$ undergoes a phase reversal whenever $f(t)$ crosses zero. The envelope of $\phi(t)$ is different from $f(t)$. Both amplitude and phase of $\phi(t)$ carry information of $f(t)$.

- The transmission bandwidth required by DSB-SC is the same as that for DSB-LC, i.e. β_T=2B.

Demodulation of DSB-SC signals

Given $\phi(t)$, how will be the message signal $f(t)$ be recovered?



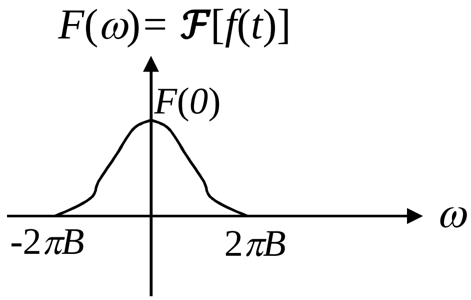
$A_c^{'}\cos(\omega_c t)$: Locally generated carrier signal

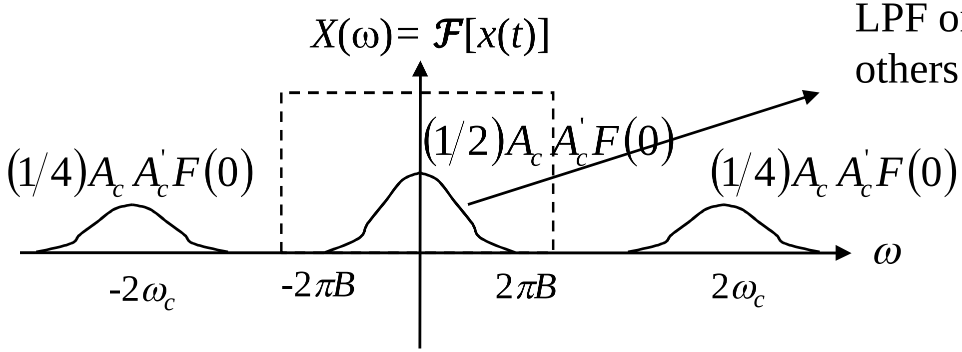
$$x(t) = \phi(t) A_c^{'} \cos(\omega_c t) = A_c^{'} A_c f(t) \cos^2(\omega_c t)$$

$$= \frac{1}{2} A_c^{'} A_c f(t) + \frac{1}{2} A_c^{'} A_c f(t) \cos(2\omega_c t)$$

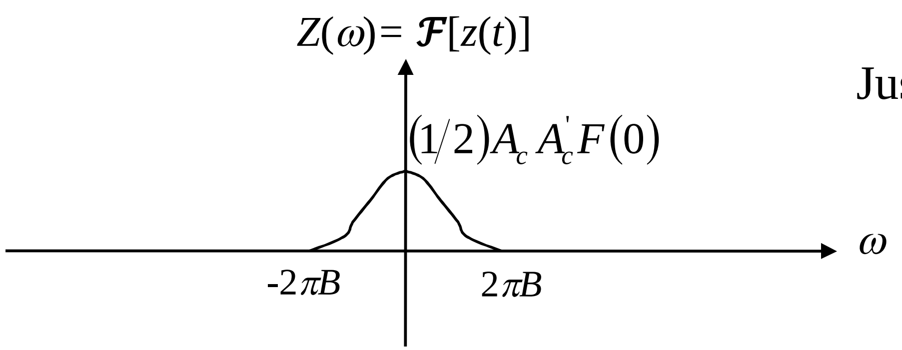
Let $X(\omega) = \mathcal{F}[x(t)]$

$$X(\omega) = \frac{1}{2} A_c A_c^{'} F(\omega) + \frac{1}{4} A_c A_c^{'} \left[F(\omega - 2\omega_c) + F(\omega + 2\omega_c) \right]$$



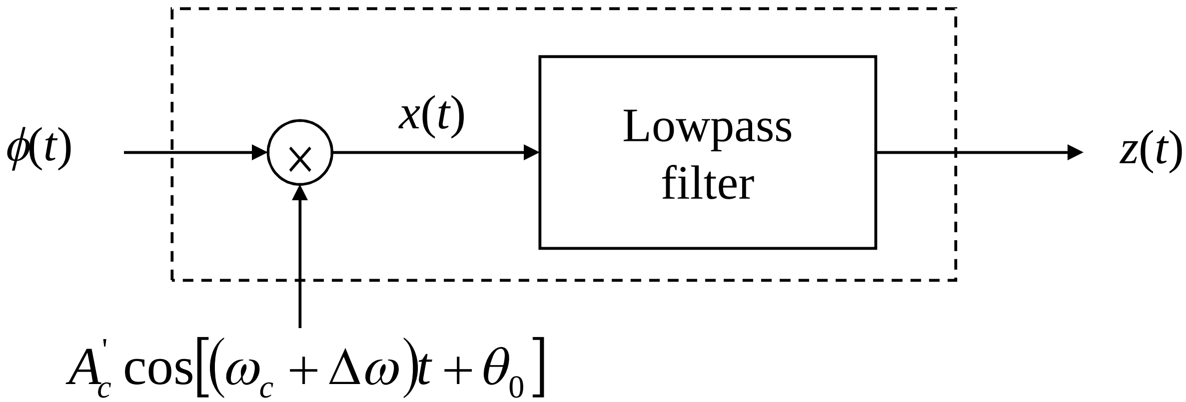


LPF only allows this component, others are rejected



Just a scaled version of $f(t)$

Now assume a frequency error and a phase error in the locally generated signal at the receiver.



$$x(t)=\phi(t)A_c^{'}\cos[(\omega_c+\Delta\omega)t+\theta_0]$$

$$=A_cA_c^{'}f(t)\cos(\omega_c t)\cos[(\omega_c+\Delta\omega)t+\theta_0]$$

$$=\underbrace{\frac{1}{2}A_cA_c^{'}f(t)\cos(\Delta\omega t+\theta_0)}+\frac{1}{2}A_cA_c^{'}f(t)\cos[(2\omega_c+\Delta\omega)t+\theta_0]$$

Only this term goes through LPF

$$z(t)=\frac{1}{2}A_c A_c^{'} f(t)\cos(\Delta\omega t+\theta_0)$$

- If $\Delta\omega$=0 and θ_0= 0, the output is $z(t)=\frac{1}{2}A_c A_c^{'} f(t)$ → no distortion

- If $\Delta\omega$ =0, the output is $z(t)=\frac{1}{2}A_c A_c^{'} f(t)\cos(\theta_0)$

  The phase error introduces a variable attenuation factor. For small fixed phase errors, this is quite tolerable. If θ_0= ±90º, the received signal is wiped out.

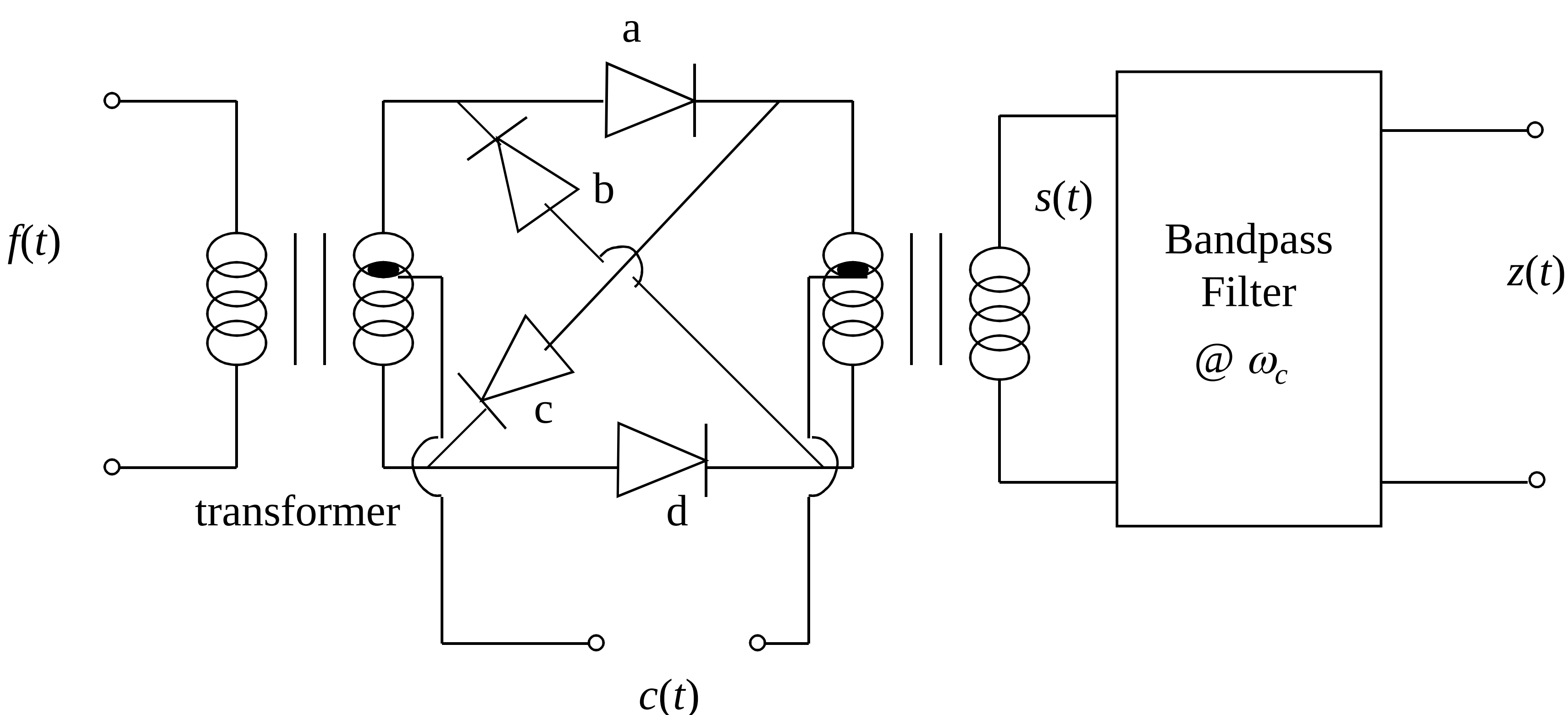
- If θ_0= 0, the output is $z(t)=\frac{1}{2}A_c A_c^{'} f(t)\cos(\Delta\omega t)$

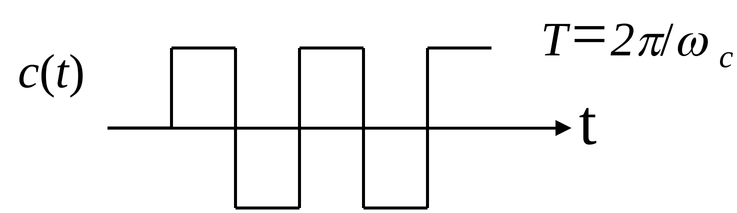
To recover $f(t)$ accurately from $\phi(t)$, we need to use a synchronized oscillator
→ *Coherent detection* (synchronous detection) is required.

Generation of DSB-SC signals: *A practical implementation*

Circuitry aspects of this topic will not be in any examination, mathematical aspects may be tested.

Ring Modulator

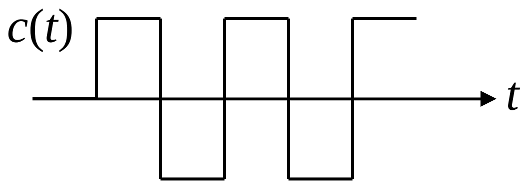




The diodes are controlled by a square-wave carrier $c(t)$ of frequency ω_c, assuming $|c(t)|>>|f(t)|$

$c(t)>0$ Diodes a and d conduct $\quad s(t)=f(t)$

$c(t)<0$ Diodes b and c conduct $\quad s(t)=-f(t)$



Expressing in terms of Fourier Series

$$c(t)=\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[\omega_c t(2n-1)]$$

$$s(t)=c(t)f(t)$$

$$s(t)=f(t)\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos[\omega_c t(2n-1)]$$

Output of BPF $\quad z(t)=\frac{4}{\pi}f(t)\cos(\omega_c t)$

Single Sideband (SSB) Modulation

(5.4 in Textbook)

Motivation: Both DSB-LC and DSB-SC occupy a bandwidth of 2*B*. How can we reduce the bandwidth requirements?

$$f(t) \leftrightarrow F(\omega)$$

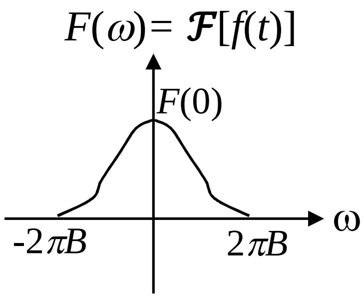
$$f^*(t) \leftrightarrow F^*(-\omega)$$

Complex conjugate property of Fourier Transform

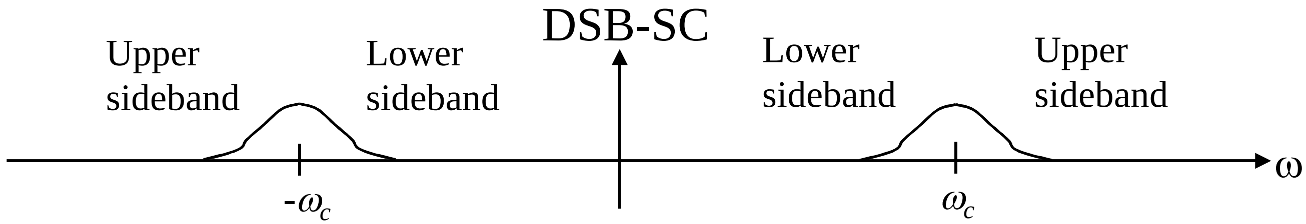
For real signals, $f(t) = f^*(t) \longrightarrow F(-\omega) = F^*(\omega)$

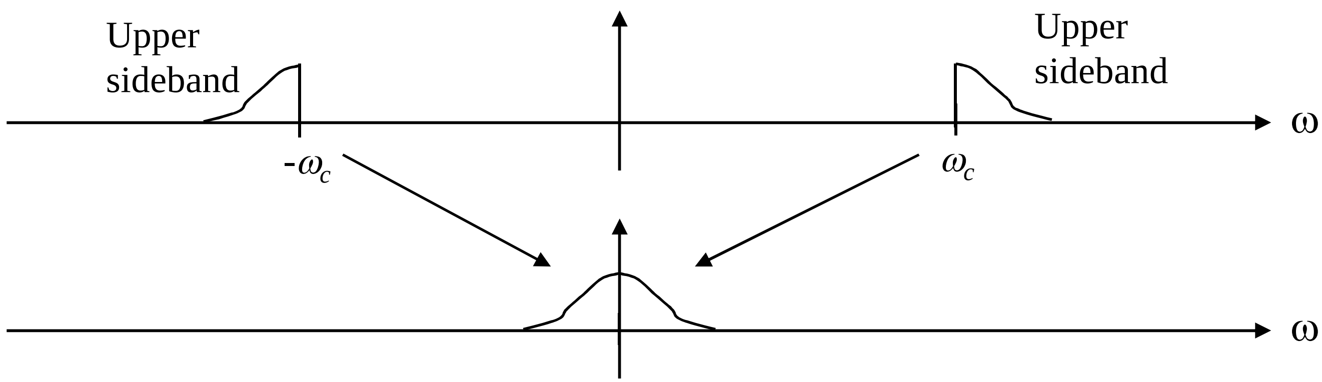
The spectral density of any <u>real-valued</u> signal exhibits the symmetry condition w.r.t. $\omega = 0$

Due to this symmetry condition, one of the sidebands is sufficient to provide the complete information in original signal.

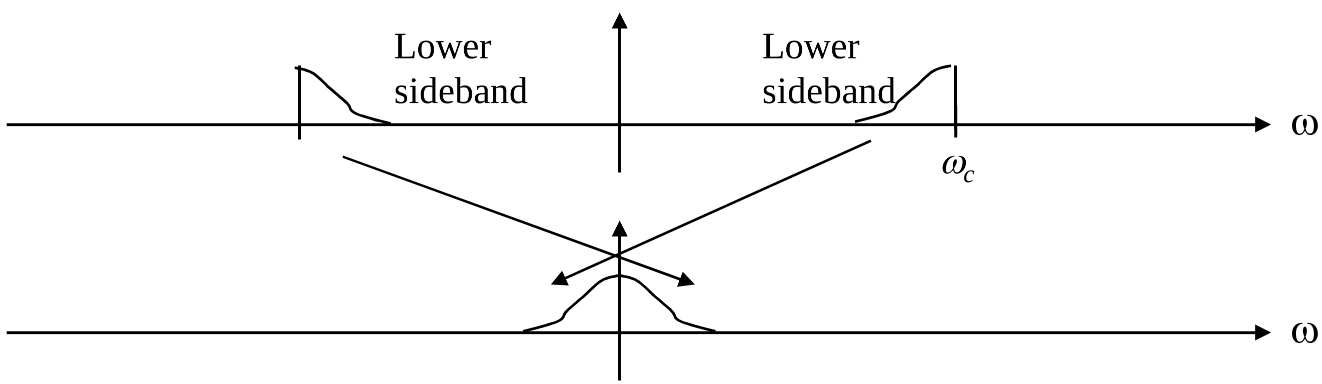


Assume similar definitions for "upper sideband" and "lower sideband" hold for negative frequencies.



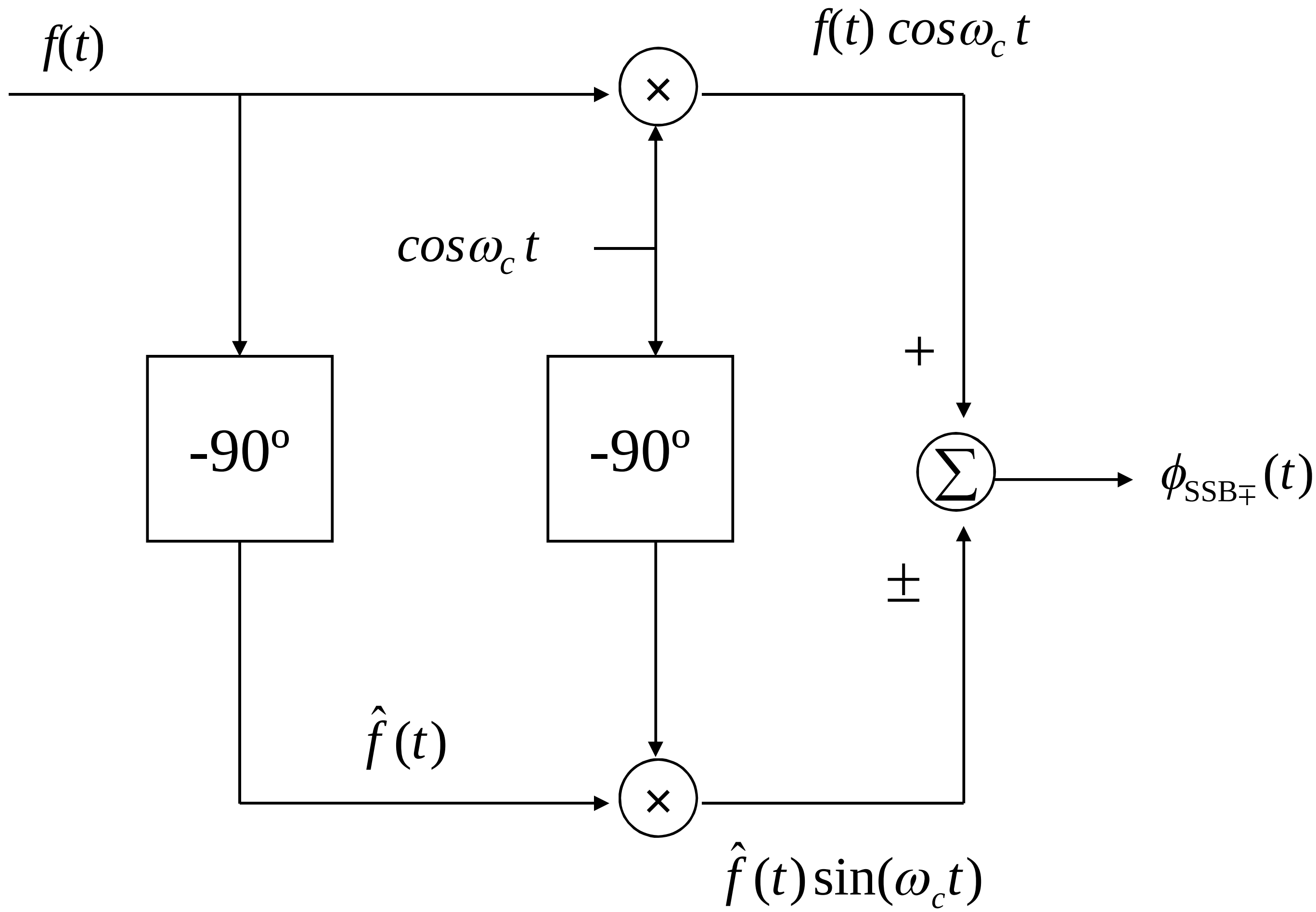


Reconstructed signal using <u>only upperband</u>



Reconstructed signal using <u>only lowerband</u>

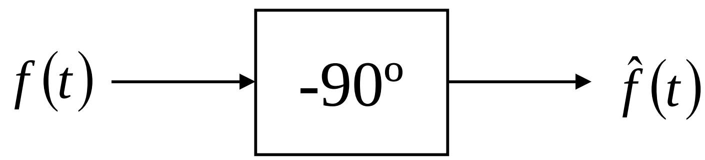
Generation of SSB Signals



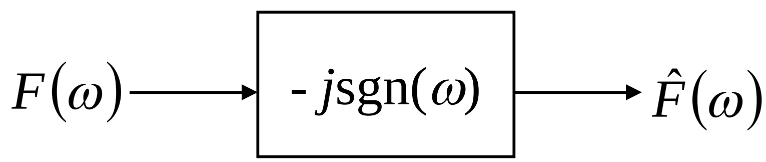
$\hat{f}(t)$ is called the *quadrature function* of $f(t)$, or the *Hilbert transform* of $f(t)$.

Hilbert Transform

Phase shifter



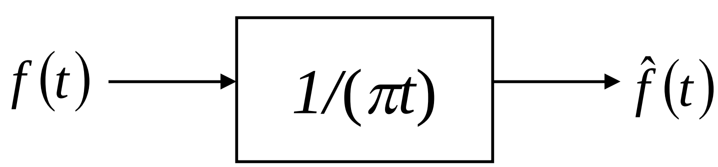
Frequency-domain description



$$\hat{F}(\omega) = -jF(\omega)\mathrm{sgn}(\omega) \qquad \mathrm{sgn}(\omega) = \begin{cases} +1, & \omega > 0 \\ -1, & \omega < 0 \end{cases}$$

Positive spectral components are shifted by -90 and negative spectral components are shifted by +90.

Time-domain description



$$\mathrm{sgn}(\omega) \leftrightarrow \frac{j}{\pi t}$$

$$\hat{f}(t) = f(t) \otimes \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau} d\tau$$

Properties of Hilbert Transform

- $f(t)$ and $\hat{f}(t)$ have the same amplitude spectral density.

$$\hat{F}(\omega) = -jF(\omega)\operatorname{sgn}(\omega) = \begin{cases} -jF(\omega), & \omega > 0 \\ +jF(\omega), & \omega < 0 \end{cases}$$

$$\longrightarrow \qquad \left|\hat{F}(\omega)\right| = \left|-jF(\omega)\right| = \left|jF(\omega)\right| = \left|F(\omega)\right|$$

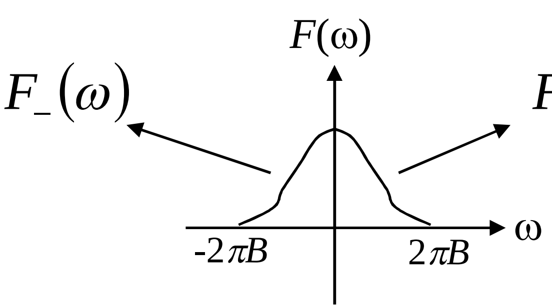
- If $\hat{f}(t)$ is the Hilbert transform of $f(t)$, the Hilbert transform of $\hat{f}(t)$ is $-f(t)$.

- $f(t)$ and $\hat{f}(t)$ are orthogonal. $\int \hat{f}(t) f(t) dt = 0$

Analytic Signals

Definition: A real-valued signal can be represented in terms of complex valued signals with one-sided spectral density. Such signals are called *analytic signals*. Its real part gives the original signal.

$$f(t) \longleftrightarrow F(\omega)$$



$$f_+(t) \longleftrightarrow \quad F_+(\omega) = \begin{cases} F(\omega), & \omega > 0 \\ 0 \quad , & \omega < 0 \end{cases}$$

$$f_-(t) \longleftrightarrow \quad F_-(\omega) = \begin{cases} 0 \quad , & \omega > 0 \\ F(\omega), & \omega < 0 \end{cases}$$

$$F(\omega) = F_+(\omega) + F_-(\omega) = F_+(\omega) + F_+^*(-\omega)$$

$$f(t) = f_+(t) + f_-(t) = f_+(t) + f_+^*(t) = 2\,\mathrm{Re}\{f_+(t)\}$$

analytic signal

Analytic Signals (Cont'd)

$$F(\omega) = F_+(\omega) + F_-(\omega)$$

Hilbert transform

$$\hat{F}(\omega) = -jF_+(\omega) + jF_-(\omega)$$

Inverse Fourier Transform

$$\hat{f}(t) = -j\ f_+(t) + j\ f_-(t)$$

$$f_-(t) = f(t) - f_+(t)$$

$$= -2j\ f_+(t) + j\ f(t)$$

$$\longrightarrow \quad f_+(t) = \underbrace{\frac{1}{2} f(t)}_{\text{Real part}} + \underbrace{j\frac{1}{2}\hat{f}(t)}_{\text{Imaginary part}}$$

Real part Imaginary part

$$\boxed{f(t) = 2\,\mathrm{Re}\{f_+(t)\} = \mathrm{Re}\{f(t) + j\ \hat{f}(t)\}}$$