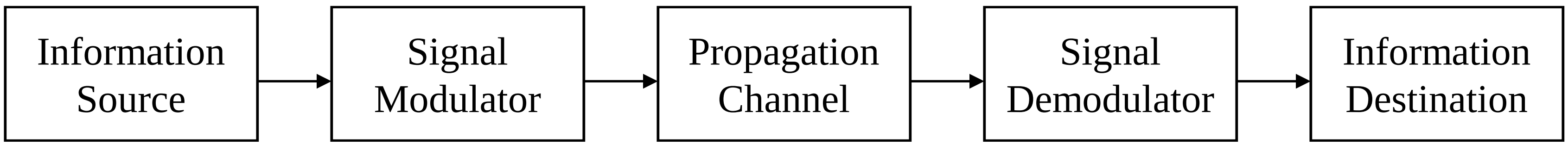
Chapter 5

Amplitude Modulation

Analog Communication System



- Analog signals may be transmitted directly via carrier modulation over the propagation channel and to be carrier-demodulated at the receiver.

Transmitter → Modulator

Receiver → Demodulator

Modulation: The process by which some characteristics of a carrier signal (i.e. modulated signal) is varied in accordance with message signal (i.e. modulating signal)

- $f(t)$: message signal

A bandlimited signal whose frequency content is in the neighbourhood of f=0 (DC) → baseband signal

- $c(t)$: the carrier signal, independent of $f(t)$

$c(t)=A_c\, cos(2\pi f_c t+\Theta_c)$

A_c : Carrier amplitude

f_c : Carrier frequency $w_c=2\pi f_c$ (radian frequency)

Θ_c : Carrier phase

$f(t)$ modulates $c(t)$ in either amplitude, frequency or phase. In effect, modulation converts $f(t)$ to a bandpass form, in the neighborhood of the center frequency f_c.

Why is Modulation Required?

- **To achieve easy radiation:** If the communication channel consists of free space, antennas are required to radiate and receive the signal. Dimension of the antennas is limited by the corresponding wavelength.

Example: Voice signal bandwidth f=3kHz

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{3 \cdot 10^3} = 10^5 \text{ m}$$

$\rightarrow$ $\lambda/4$=25000m!!

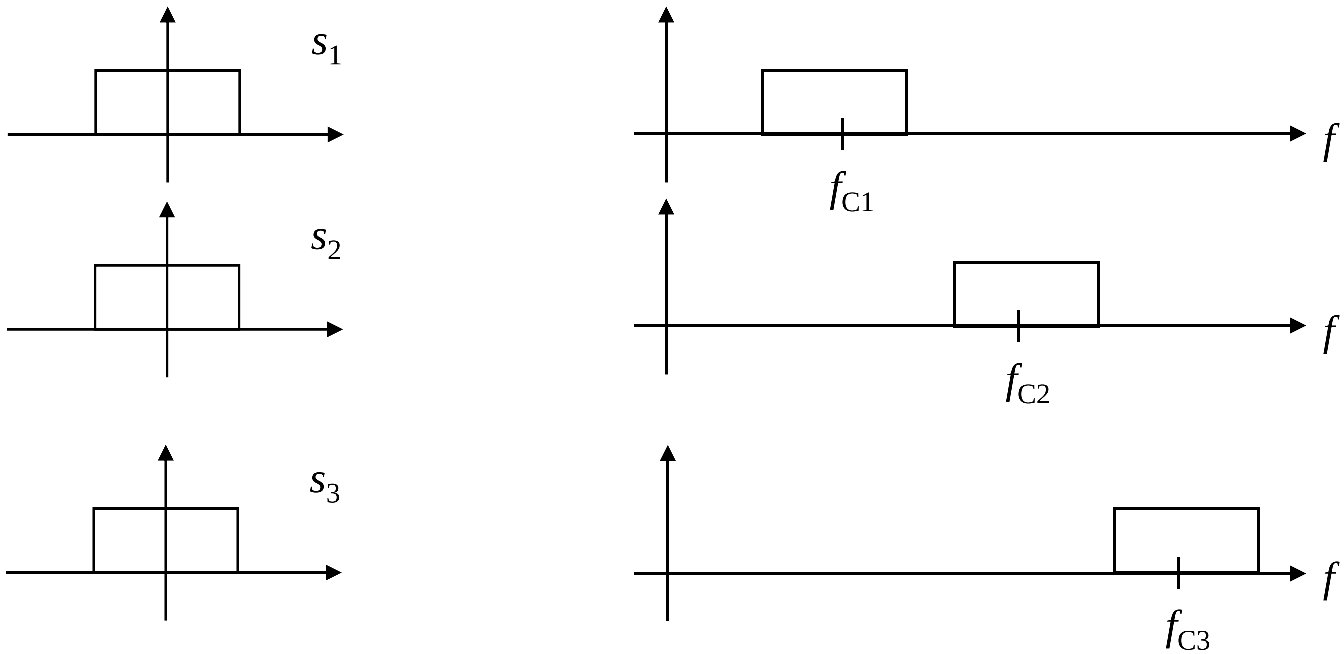
If we modulate a carrier wave @ f_c = 100MHz with the voice signal

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8}{100 \cdot 10^6} = 3 \text{ m}$$

$\rightarrow$ $\lambda/4$=75cm

Why is Modulation Required? (Cont’d)

- **<u>To accommodate for simultaneous transmission of several signals</u>**



Example: Radio/TV broadcasting

Why is Modulation Required? (Cont'd)

- To expand the bandwidth of the transmitted signal for better transmission quality (to reduce noise and interference)

$$C = B \cdot \log_2 (1 + SNR)$$

Channel capacity   Bandwidth   Signal-to-noise ratio

Channel capacity: Maximum achievable information rate that can be transmitted over the channel

$$SNR = 2^{\frac{C}{B}} - 1$$

$B \uparrow$ The required SNR (for fixed noise level, corresponding signal power) decreases

Amplitude Modulation (AM)

(Ch. 5 in Textbook)

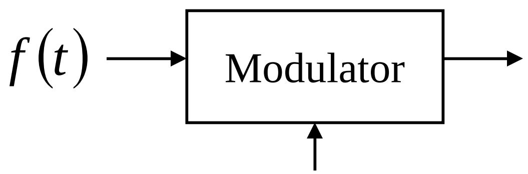
Objectives:

- To study different amplitude modulation scheme
- To study generation and detection of AM signals
- To study application of AM

We will study

- **Double Sideband Large Carrier (DSB-LC) Modulation:** Commercial broadcast stations use this type and it is commonly known as just amplitude modulation (AM).
- **Double Sideband Suppressed Carrier (DSB-SC) Modulation**
- **Single Sideband (SSB) Modulation**
- **Vestigial Sideband (VSB) Modulation**

Double Side Band Large Carrier (DSB-LC) (5.2 in Textbook)



$$c(t) = A_c \cos\omega_c t$$

$$\phi(t) = (f(t) + A_c)\cos\omega_c t$$
$$= f(t)\cos\omega_c t + A_c \cos\omega_c t$$

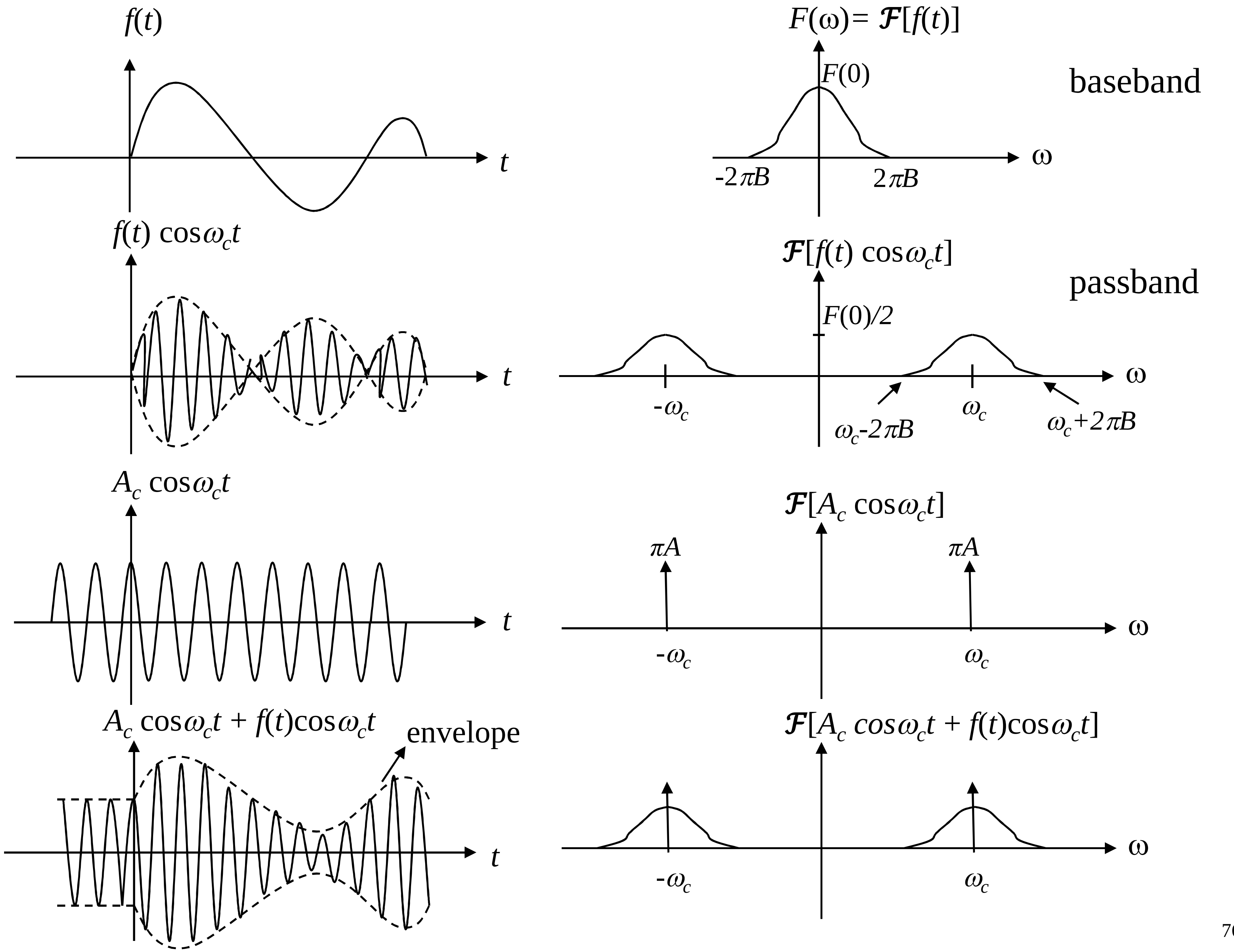
$$F(\omega) = \mathcal{F}\{f(t)\} \quad \Phi(\omega) = \mathcal{F}\{\phi(t)\}$$

$$\Phi(\omega) = \mathcal{F}\left[A_c \cos(\omega_c t) + f(t)\cos(\omega_c t)\right]$$

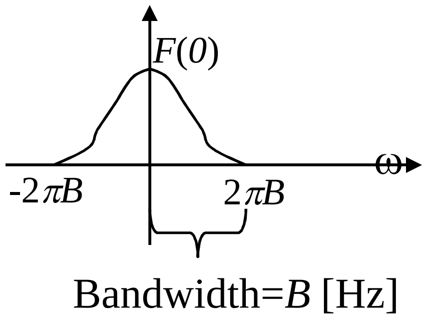
$$= \mathcal{F}\left(\frac{A_c}{2}e^{j\omega_c t} + \frac{A_c}{2}e^{-j\omega_c t} + \frac{f(t)}{2}e^{j\omega_c t} + \frac{f(t)}{2}e^{-j\omega_c t}\right)$$

$$\cos\omega_c t = \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$$

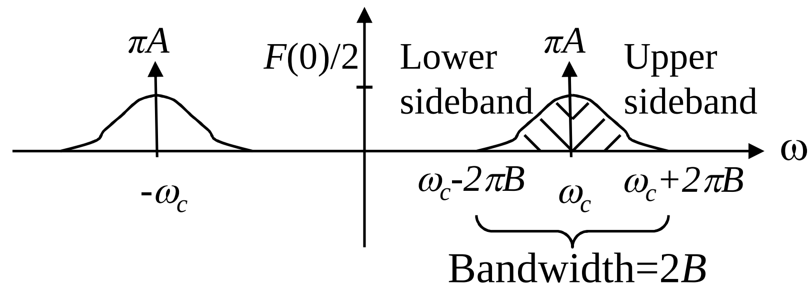
$$= \pi A_c \delta(\omega - \omega_c) + \pi A_c \delta(\omega + \omega_c) + \frac{1}{2}F(\omega - \omega_c) + \frac{1}{2}F(\omega + \omega_c)$$


f(t)
t
F(ω)= ℱ[f(t)]
F(0)
baseband
ω
-2πB
2πB
f(t) cosω_c t
ℱ[f(t) cosω_c t]
passband
F(0)/2
-ω_c
ω_c
ω_c-2πB
ω_c+2πB
A_c cosω_c t
ℱ[A_c cosω_c t]
πA
πA
A_c cosω_c t + f(t)cosω_c t
envelope
ℱ[A_c cosω_c t + f(t)cosω_c t]

baseband $F(\omega) = \mathcal{F}[f(t)]$



passband $\Phi(\omega) = \mathcal{F}[A_c \cos\omega_c t + f(t)\cos\omega_c t]$

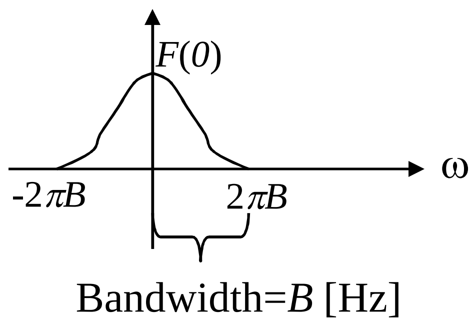


Observations:

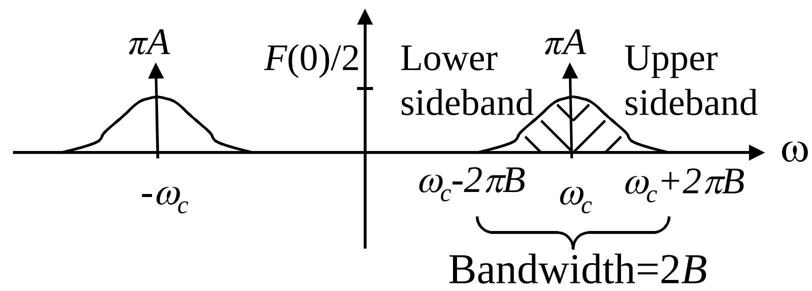
- Modulation shifts the content of $F(\omega)$ to the neighbourhood of ω_c .
- $F(\omega)$ for $\omega \in [-2\pi B, 0]$ is shifted to $\Phi(\omega)$ for $\omega \in [\omega_c - 2\pi B, \omega_c]$ and called as *lower sideband*.
- $F(\omega)$ for $\omega \in [0, 2\pi B]$ is shifted to $\Phi(\omega)$ for $\omega \in [\omega_c, \omega_c + 2\pi B]$ and called as *upper sideband*.
- Let B denote the highest frequency component of $f(t)$.

Assume $f_c >> B \rightarrow \phi(t)$ is defined as a *narrowband* signal (i.e. its spectral content is located in the immediate vicinity of some high center frequency)

$F(\omega) = \mathcal{F}[f(t)]$

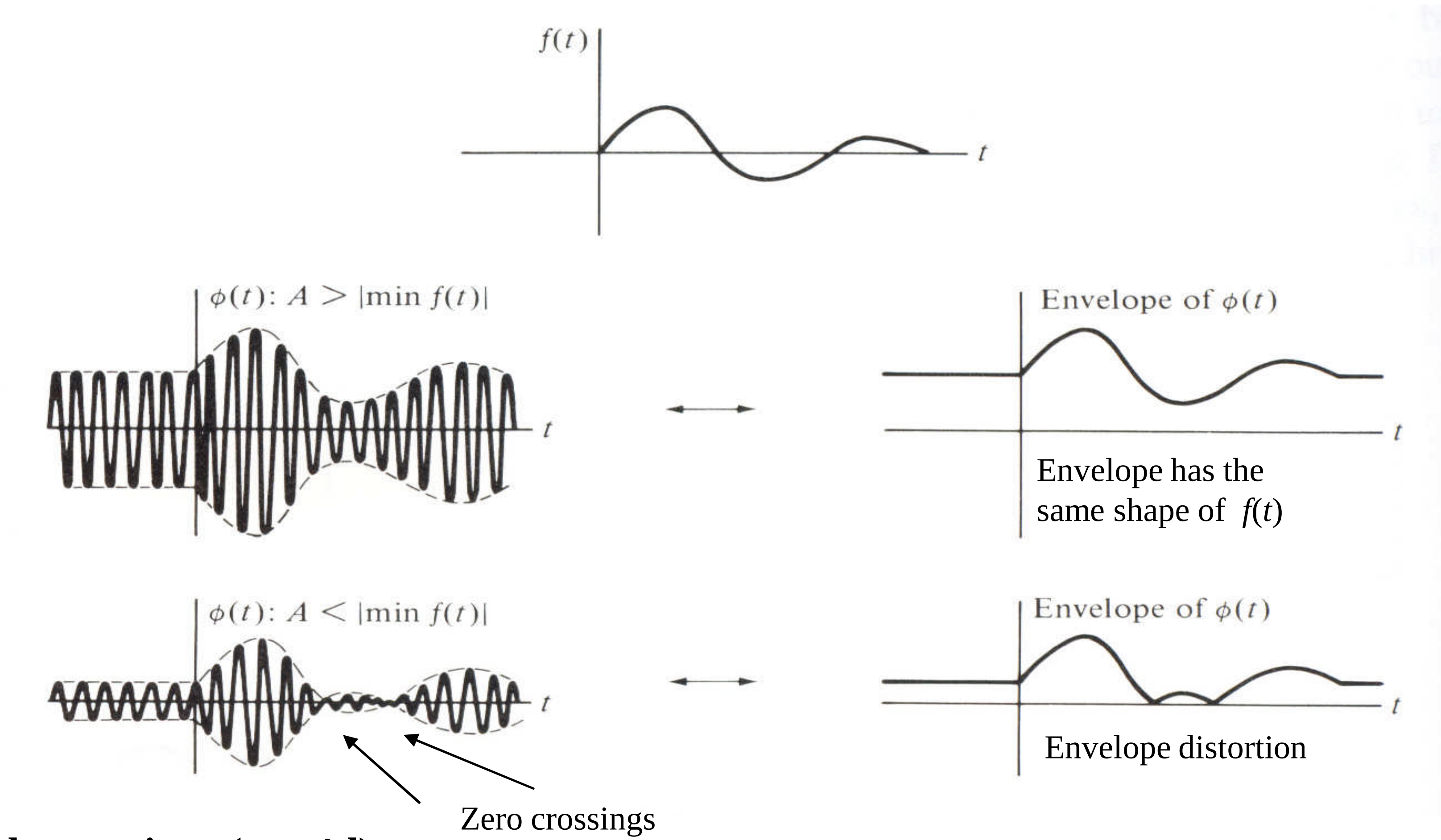


$\Phi(\omega) = \mathcal{F}[A_c \cos\omega_c t + f(t)\cos\omega_c t]$



Observations (cont’d)

- The bandwidth of message signal is B. The transmission bandwidth $\beta_T = 2B$ (i.e. DSB-LC is wasteful of bandwidth)
- The carrier term does not carry any information and hence the carrier power is wasted.



Observations (cont'd)

- If $A_c+f(t)>0$ for all t, the envelope of $\phi(t)$ of has essentially the same shape as the $f(t)$.
- If $A_c+f(t)<0$ for any t, the carrier wave becomes *over-modulated*, resulting in carrier phase reversal whenever $A_c+f(t)$ crosses zero.

  $\rightarrow$ $\phi(t)$ has *envelope distortion*

Example: Single-frequency sinusoidal tone as modulating signal

$f(t) = A_m \cos(\omega_m t)$, $c(t) = A_c \cos(\omega_c t)$

$\phi(t) = [A_c + A_m \cos(\omega_m t)] \cos\omega_c t$

Define modulation index $m = A_m / A_c$

$\phi(t) = A_c[1 + m \cos(\omega_m t)] \cos\omega_c t$

Let A_{max} and A_{min} denote the maximum and minimum values of envelope of $\phi(t)$, then

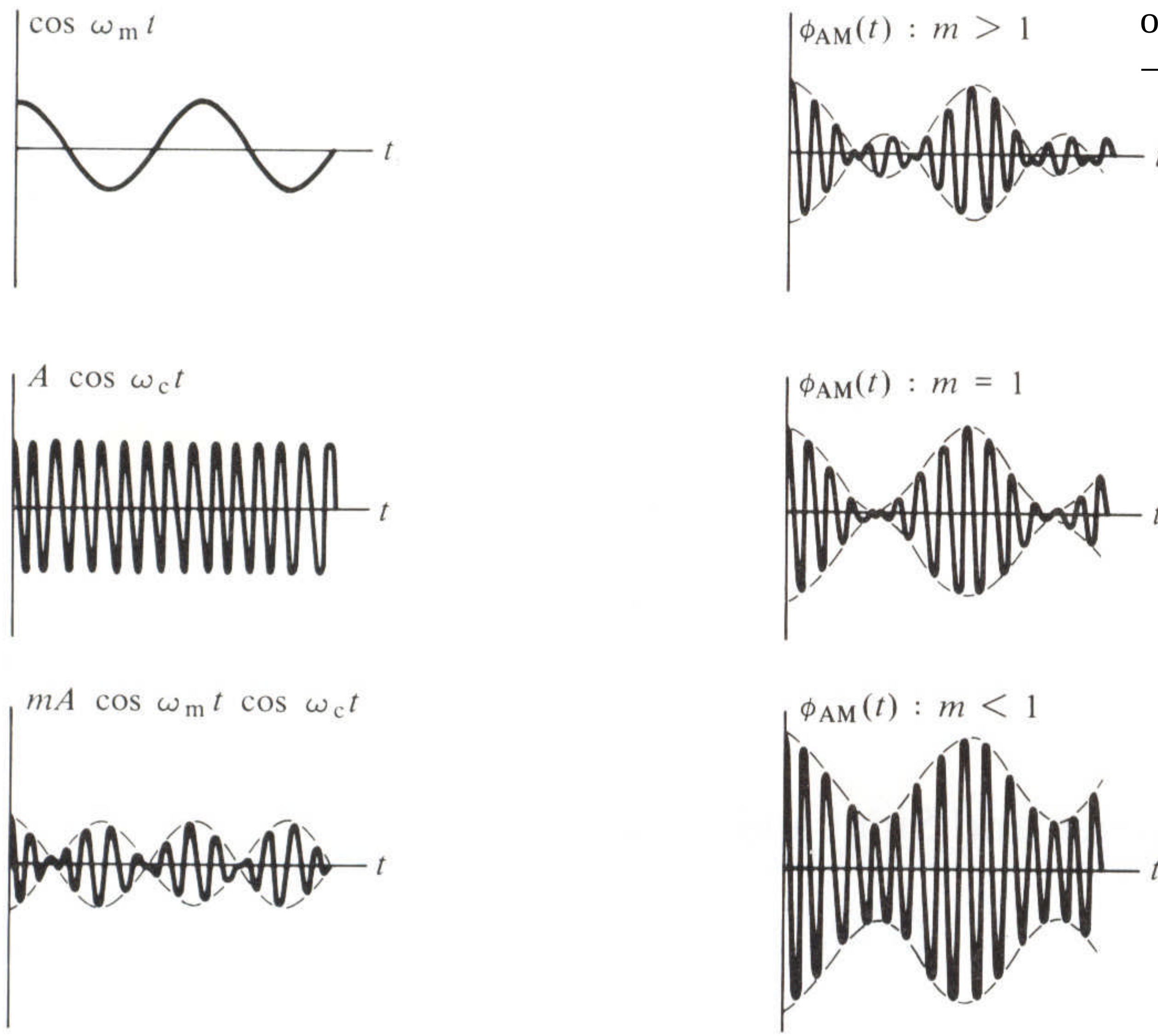
$A_{max} = A_c(1+m)$, $A_{min} = A_c(1-m)$

$A_{max} / A_{min} = (1+m)/(1-m) \rightarrow \quad m = (A_{max} - A_{min})/(A_{max} + A_{min})$

Since $\cos(\omega_m t) \cos(\omega_c t) = (1/2) \cos[(\omega_c + \omega_m) t] + (1/2) \cos[(\omega_c - \omega_m) t]$

$\phi(t) = A_c \cos(\omega_c t) + (m/2) A_c \cos[(\omega_c + \omega_m) t] + (m/2) A_c \cos[(\omega_c - \omega_m) t]$

overmodulated
→ envelope dist.



Effects of varying modulation indexes

Carrier and Sideband Power in DSB-LC

$$\phi(t) = A_c \cos(\omega_c t) + f(t)\cos(\omega_c t)$$

$$\overline{\phi^2(t)} = A_c^2\,\overline{\cos^2(\omega_c t)} + \overline{f^2(t)\cos^2(\omega_c t)} + 2A_c\,\overline{f(t)\cos^2(\omega_c t)}$$

Assume $\overline{f(t)} = 0$ and $f(t)$ varies slowly with respect to $\cos(\omega_c t)$

$$\overline{\phi^2(t)} = A_c^2\,\overline{\cos^2(\omega_c t)} + \overline{f^2(t)}\;\overline{\cos^2(\omega_c t)}$$

$$\overline{\phi^2(t)} = A_c^2/2 + \overline{f^2(t)}/2$$

$$\overline{\cos^2(\omega_c t)} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}\cos^2\omega_c t\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}\frac{1}{2}(1+\cos 2\omega_c t)dt$$

$$= \frac{1}{2}\lim_{T\to\infty}\frac{t}{T}\bigg|_{-T/2}^{T/2} + \underbrace{\frac{1}{2}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2}(1+\cos 2\omega_c t)dt}_{=0} = \frac{1}{2}$$

$$\overline{\phi^2(t)} = A_c^2/2 + \overline{f^2(t)}/2$$

Carrier Power

Sideband Power (carries information)

Modulation (Power) Efficiency

$$\mu = \frac{\text{useful power}}{\text{total power}}$$

$$= \frac{\overline{f^2(t)}/2}{A_c^2/2 + \overline{f^2(t)}/2}$$

Example (Cont'd)

$$\phi(t) = A_c \cos(\omega_c t) + \frac{mA_c}{2}\cos[(\omega_c + \omega_m)t] + \frac{mA_c}{2}\cos[(\omega_c - \omega_m)t]$$

Upper sideband power $\left(\frac{mA_c}{2}\right)^2 \overline{\cos^2[(\omega_c + \omega_m)t]} = \frac{m^2 A_c^2}{8}$

Lower sideband power $\left(\frac{mA_c}{2}\right)^2 \overline{\cos^2[(\omega_c - \omega_m)t]} = \frac{m^2 A_c^2}{8}$

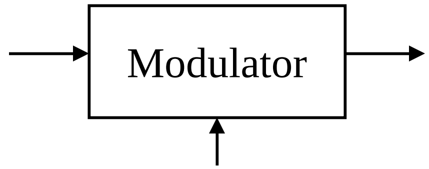
$$\mu = \frac{\text{total useful power}}{\text{total power}} = \frac{m^2 A_c^2 / 4}{A_c^2/2 + m^2 A_c^2/4} = \frac{m^2}{2 + m^2}$$

For $m \leq 1 \rightarrow \mu \leq 33\%$. Under the best condition, i.e. m=1, 67% of the total power is used in the carrier and represents wasted power.

Generation of DSB-LC Signals

Circuitry aspects of this topic will not be in any examination, mathematical aspects may be tested.

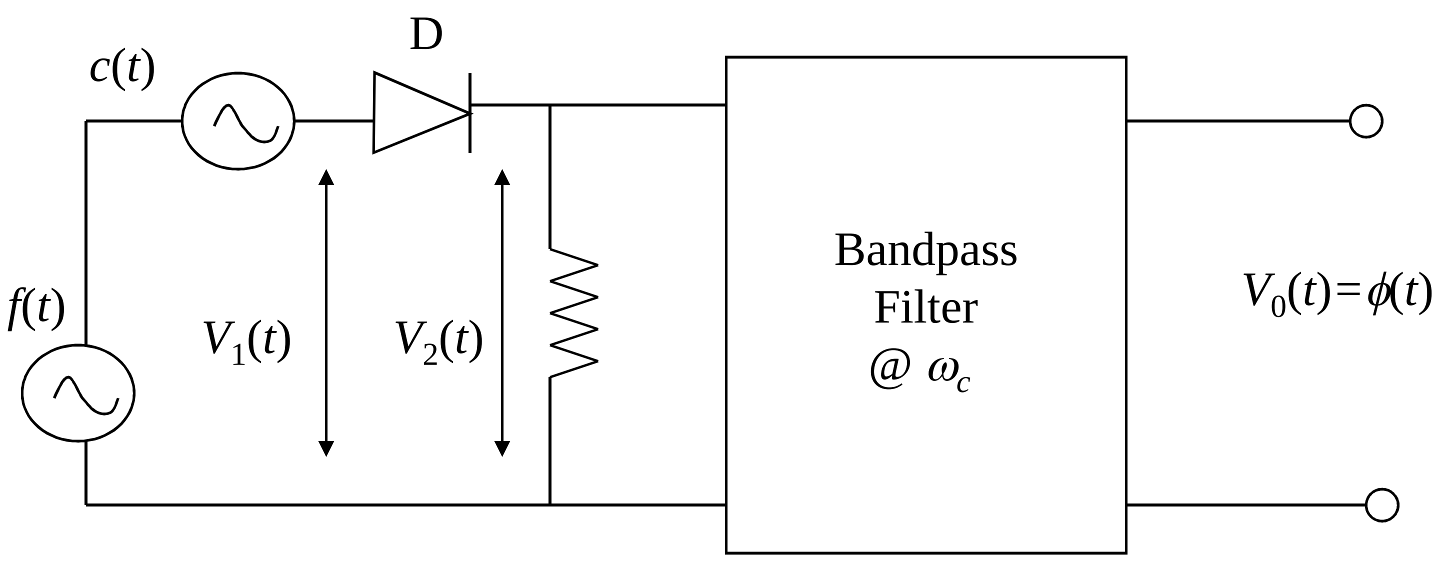
Given $f(t)$, how will be the modulated signal $\phi(t)$ be generated?



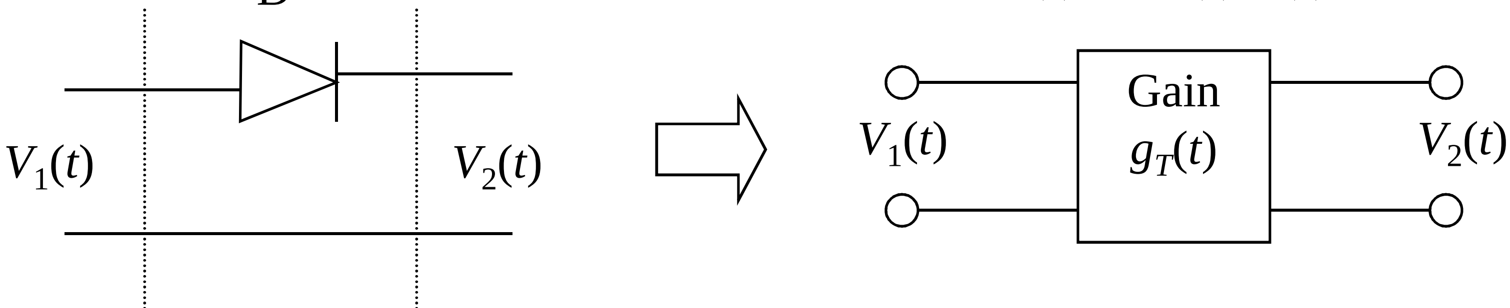
$$f(t) \rightarrow \text{Modulator} \rightarrow \phi(t) = f(t)\cos\omega_c t + A_c \cos\omega_c t$$

$$c(t) = A_c \cos\omega_c t$$

Chopper Modulator



$$V_2(t) = g_T(t)V_1(t)$$



$$V_1(t) = f(t) + c(t) = f(t) + A_c \cos(\omega_c t)$$

Assume D is an ideal diode, and $|f(t)| < A_c$, then

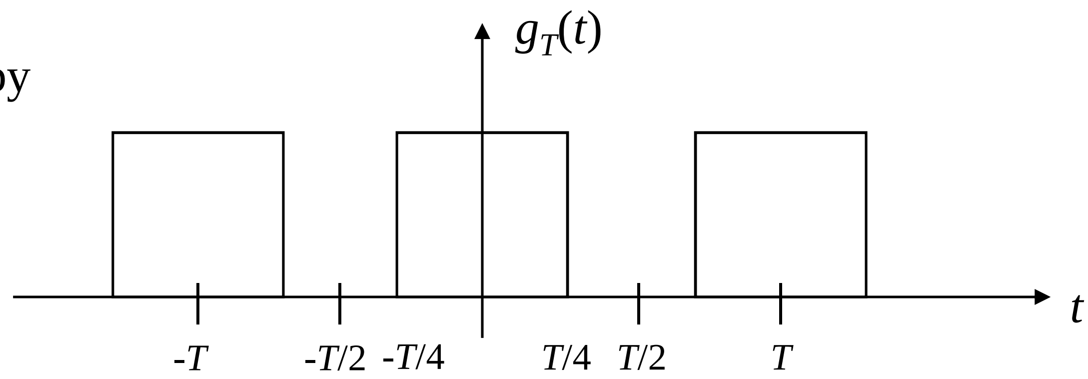
$$V_2(t) = \begin{cases} V_1(t), & \text{if } c(t) > 0 \\ 0, & \text{if } c(t) < 0 \end{cases}$$

i.e. the load voltage $V_2(t)$ varies periodically between the values $V_1(t)$ and 0 at a rate equal to the carrier frequency ω_c .

When the diode is on, $g_T(t) = 1$
is off, $g_T(t) = 0$
} chopper modulation

The diode can be represented by a rectangle pulse generator.

$T = 2\pi/\omega_c = 1/f_c$



Expand $g_T(t)$ using Fourier Series, we have

$$g_T(t) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(\omega_c t(2n-1))$$

$$V_2(t) = V_1(t) g_T(t)$$

$$V_2(t) = (f(t) + A_c\cos(\omega_c t))\left\{\frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(\omega_c t(2n-1))\right\}$$

$$= \frac{1}{2}f(t) + \frac{1}{2}A_c\cos(\omega_c t) + \frac{2}{\pi}\cos^2(\omega_c t) + \frac{2}{\pi}f(t)\cos(\omega_c t)$$

$$- \frac{2}{3\pi}A_c\cos(\omega_c t)\cos(3\omega_c t) - \frac{2}{3\pi}f(t)\cos(3\omega_c t) + \text{other terms}$$