2. Double-Sideband Suppressed Carrier Modulation (DSB-SC)

In full AM (DSB-AM), the carrier wave c(t) is completely independent of the message signal m(t), which means that the transmission of carrier wave represents a **waste of power**.
This points to a shortcoming of amplitude modulation, that only a fraction of the total transmitted power is affected by m(t) as seen in page 22

$$[\eta(efficiency) = \frac{\mu^2}{1+\mu^2}]$$

$$\eta = 33.3\% \text{ when } \mu = 1$$

$$\eta < 33.3\% \text{ when } \mu < 1$$

To overcome this shortcoming, we may suppress the carrier component from the modulated wave, resulting in double sided-suppressed carrier (DSB-SC) modulation.
A double-sideband, suppressed carrier AM signal is obtained by multiplying the message signal m(t) with the carrier signal.
Thus we have the amplitude modulated signal

$$s(t) = c(t).m(t) \qquad (23).$$

$$= \underbrace{A_c \cos(\omega_c t)}_{\text{Carrier Signal}}.\underbrace{A_m \cos(\omega_m t)}_{\text{Modulating Signal}} \qquad (24).$$

$$= A_m A_c \cos(\omega_c t)\cos(\omega_m t)$$

$$= A_m A_c [\frac{\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t}{2}]$$

$$\boxed{\Rightarrow \underbrace{s(t)}_{\text{DSB-SC Wave}} = \frac{A_m A_c}{2}\underbrace{\cos(\omega_c + \omega_m)t}_{\text{Upper Side Frequency}} + \frac{A_m A_c}{2}\underbrace{\cos(\omega_c - \omega_m)t}_{\text{Lower Side Frequency}}} \qquad (25).$$

(But No Carrier Here)

Note: Compare equations (12) (DSB-AM) and equations (25) (DSB-SC)

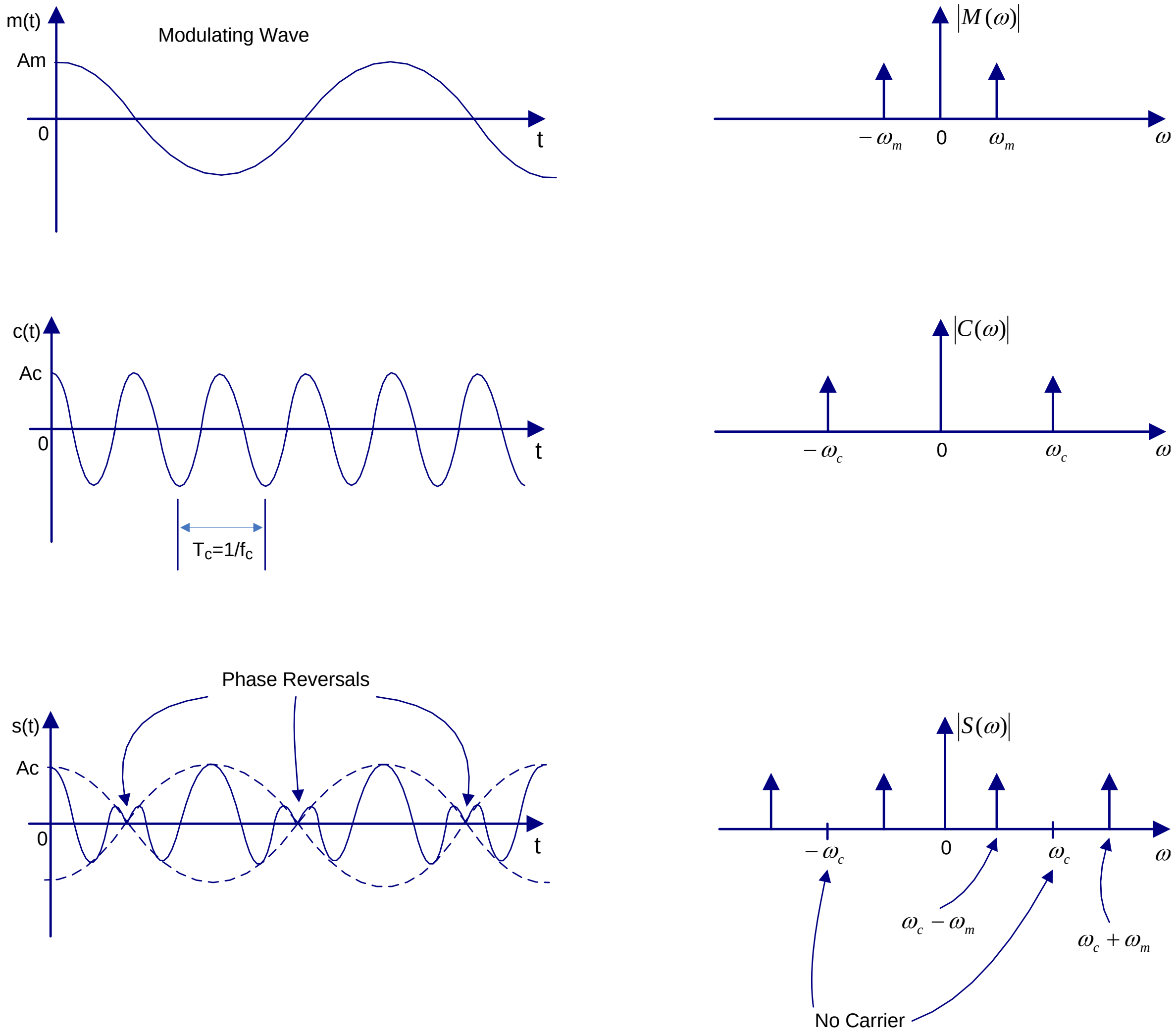


Fig 11.

The spectrum of DSB-SC modulated wave consists of impulse functions located at $\omega_c \pm \omega_m$ and $-\omega_c \pm \omega_m$.

Note: Suppression of the carrier has a profound impact on the waveform of the modulated signal and its spectrum. (See Fig3. page 19 for DSB (full) and Fig 11. page 29 for DSB-SC)

When the message signal m(t) is limited to the interval $-\omega_m \le \omega \le \omega_m$ (not a single tone but it is a spectrum) the DSB-SC modulation process simply translates the spectrum of the message signal by $\pm\omega_c$ (See Fig 12.).

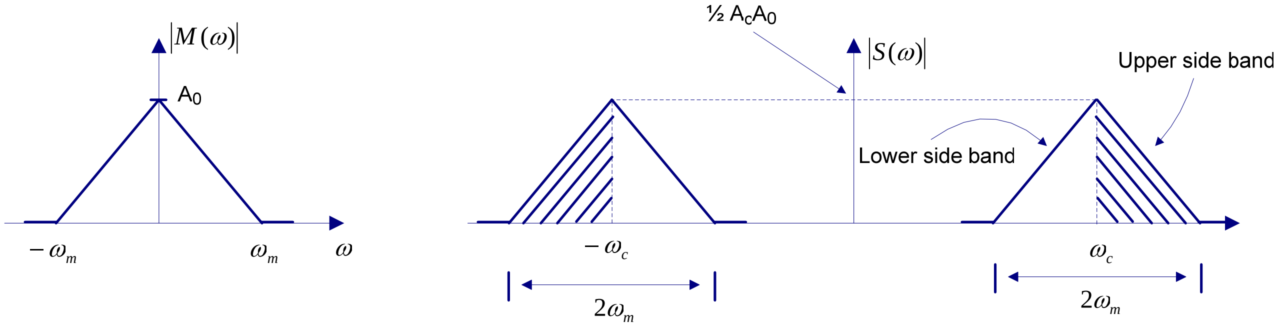


Fig 12.

The transmission bandwidth required for DSB-SC modulation is the same as that for full amplitude modulation, namely $2\omega_m$.

The generation of a DSB-SC modulated wave consists simply of the product of the message signal m(t) and the carrier wave $A_c \cos(\omega_c t)$. A device for achieving this requirement is called a **product modulator**, which is another term for a straightforward multiplier as shown in Fig 13.

Generation of DSB-SC Wave

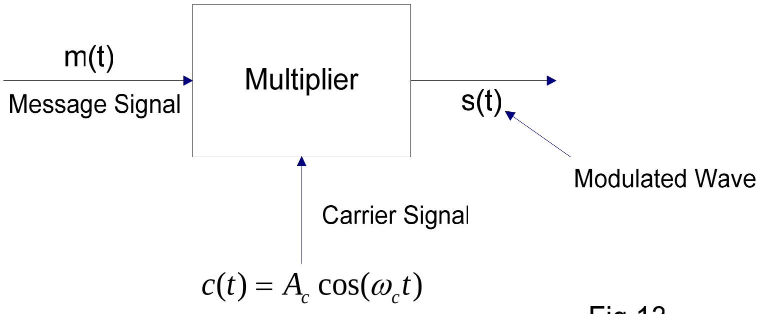


Fig 13.

Power content of the DSB-SC Wave

$$s(t) = \frac{A_m A_c}{2}\cos(\omega_c + \omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c - \omega_m)t$$

(See equation 25)

Modulated Wave
(DSB-SC)

Average power delivered to a 1Ω resistor can be calculated as,

Upper side frequency Power (Pu) = $(\frac{A_m A_c/2}{\sqrt{2}})^2$

$$\boxed{P_u = \frac{A_m{}^2 A_c{}^2}{8}} \qquad (28).$$

$$\text{Lower side frequency Power } (\mathrm{P_L}) = (\frac{A_m A_c/2}{\sqrt{2}})^2$$

$$\boxed{P_L = \frac{A_m{}^2 A_c{}^2}{8}} \qquad (29).$$

$$\text{Total Power } (P_T) = P_u + P_L = 2(\frac{A_m{}^2 A_c{}^2}{8}) = \frac{A_m{}^2 A_c{}^2}{4}$$

in the DSB-SC
modulated wave

$$\therefore \frac{P_u}{P_T} = \frac{P_L}{P_T} = \frac{A_m{}^2 A_c{}^2/8}{A_m{}^2 A_c{}^2/4} \times 100 = 50\%$$

$\therefore$ For the sinusoidal modulation, the average power in the lower or upper side-frequency with respect to the total power in the DSB-SC modulated wave is 50%.

Demodulation of DSB-SC AM signals

The message signal m(t) may be recovered from a DSB-SC modulated wave s(t) with a locally generated sinusoidal wave and then low-pass filtering the product as shown in Fig 14.

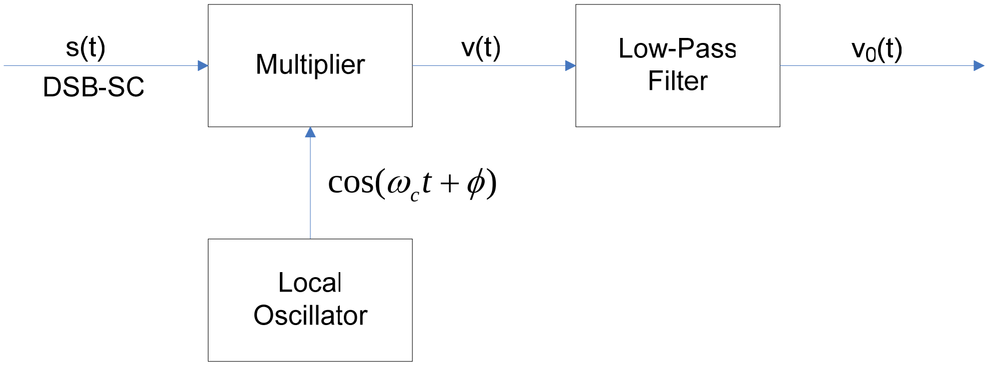


Fig 14.

It is **assumed** that the local oscillator output is exactly synchronised, in both frequency and phase with the carrier wave c(t) used in the product modulator to generate s(t). [See Fig 13.]

This method of demodulation is called “Coherent Detection”.

Let us assume that the local oscillator frequency at the receiver is the same frequency as the carrier, but arbitrary phase difference ϕ, measured with respect to the carrier wave c(t).

∴ We know that $s(t) = A_c \cos\omega_c t.m(t)$ {See equation 24, page 29}

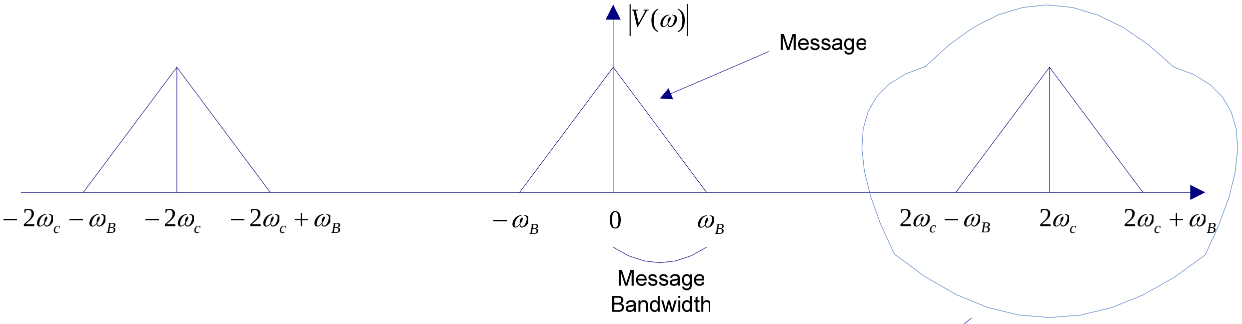
∴ $v(t) = s(t)\cos(\omega_c t+\phi) = A_c \cos\omega_c t.m(t)\cos(\omega_c t+\phi)$

(multiplier output, Fig 14.)

Scaling Factor

$$\Rightarrow v(t) = \underbrace{\boxed{\frac{1}{2}A_c\cos(\phi)}.m(t)}_{\text{This represents a scaled version of the original signal m(t).}} + \underbrace{\frac{1}{2}A_c\cos(2\omega_c t+\phi).m(t)}_{\text{This represents a new DSB-SC modulated wave with carrier frequency } 2\omega_c.} \qquad (30).$$

Figure 15 shows the magnitude spectrum of v(t).



We assume $\omega_c >> \omega_B$ where ω_B is the message bandwidth.

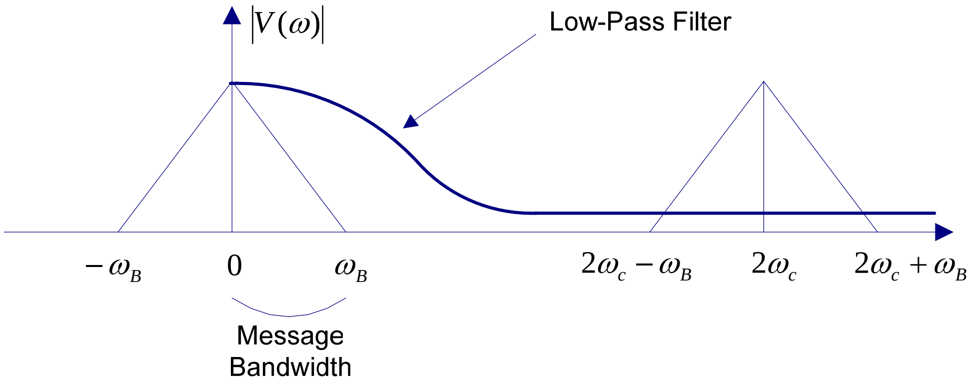
We may use a low-pass filter to suppress the unwanted term $\frac{1}{2}A_c\cos(2\omega_c t+\phi)m(t)$ of v(t).

Fig 15.

To accomplish this, the pass-band of the low-pass filter must extend over the entire message spectrum. More precisely, the low-pass filter specifications must satisfy two requirements:

1. Cutoff frequency : ω_B
2. Transmission band : $\omega_B \le \omega \le 2\omega_c - \omega_B$

Thus the overall output $v_0(t)$ (See Fig 14.) is given by $\boxed{v_0(t) = \frac{1}{2}A_c\cos\phi.m(t)}$ (31).



From equation 31, we see:

- The modulated signal $v_0(t)$ is proportional to m(t) when the phase error ϕ is a constant.($v_0(t)\ \alpha\ m(t)$)
- The amplitude of the demodulated signal $v_0(t)$ is maximum when $\phi = 0 \Rightarrow \cos\phi = 1$ and has a minimum of zero when $\phi = \pm\frac{\pi}{2} \Rightarrow \cos\phi = 0$.

$$v_0(t) = \frac{1}{2} A_c \underset{\substack{\nearrow \\ \text{phase error}}}{\cos(\phi)}.\underset{\substack{\nwarrow \\ \text{message}}}{m(t)}$$

The phase error ϕ in the local oscillator causes the detector output to be attenuated by a factor equal to $\cos\phi$.
As long as the phase error ϕ is constant, the detector output provides an undistorted version of the original signal m(t).

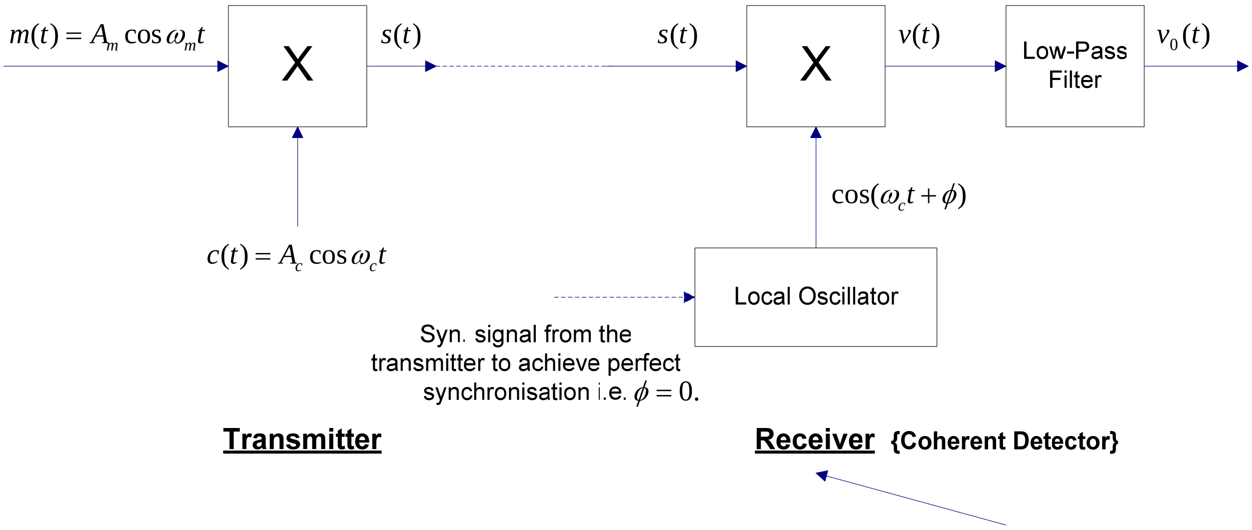
In practice, we find that the phase error ϕ varies randomly with time, owing to random variations in the communication channel.

The result is that at the detector output, the multiplying factor $\cos\phi$ also varies randomly with time, which is obviously undesirable.

Therefore circuitry must be provided at the receiver to maintain the local oscillator in perfect synchronism, in both frequency and phase, with the carrier wave used to generate the DSB-SC modulated wave in the transmitter.

The resulting increase in receiver complexity is the price that must be paid for suppressing the carrier wave to **save transmitter power**.

Analysis of DSB-SC modulation for a sinusoidal modulating wave:



When there is perfect synchronism between the local oscillator in the receiver and the carrier wave c(t) in the transmitter, we find $\phi = 0$.

$$\therefore \quad v(t) = \underbrace{A_m \cos\omega_m t \times A_c \cos(\omega_c t)}_{= \text{s(t)}} \times \cos(\omega_c t)$$

$$v(t) = [\frac{A_m A_c}{2}\cos(\omega_c - \omega_m)t + \frac{A_m A_c}{2}\cos(\omega_c + \omega_m)t] \times \cos\omega_c t$$

$$= \quad \boxed{\frac{A_m A_c}{2}\cos(\omega_c + \omega_m)t.\cos\omega_c t} \quad + \quad \boxed{\frac{A_m A_c}{2}\cos(\omega_c - \omega_m)t.\cos\omega_c t}$$

$$\Rightarrow v(t) = \frac{1}{4}A_m A_c \cos(2\omega_c - \omega_m)t + \frac{1}{4}A_m A_c \cos\omega_m t + \frac{1}{4}A_m A_c \cos(2\omega_c + \omega_m)t + \frac{1}{4}A_m A_c \cos\omega_m t$$

The frequencies $2\omega_c + \omega_m$ & $2\omega_c - \omega_m$ are removed by the low-pass filter above.

$\therefore$ The coherent detector output thus produces the original modulating wave.

$$v_0(t) = \underbrace{\frac{1}{4}A_cA_m\cos\omega_m t + \frac{1}{4}A_cA_m\cos\omega_m t}$$

After filtering v(t)

Note that the detector output appears as equal terms, one derived from the upper side-frequency and the other from the lower side-frequency.

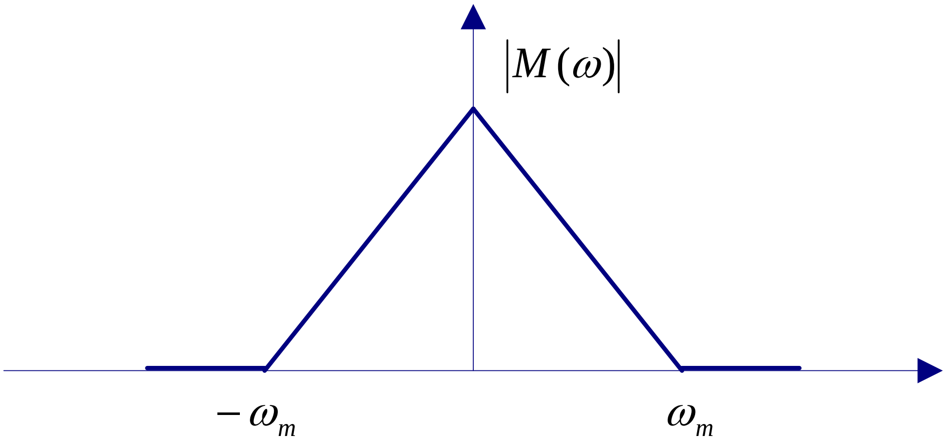
We therefore conclude that for the transmission of information, only one side-frequency is necessary. Hence Single Sideband Modulation (SSB) i.e. Next Section.

Example:

Consider a message signal m(t) with spectrum shown below. The message bandwidth $\omega_m = 2\pi \times 10^3 \text{ rad/sec}$. This signal is applied to a product modulator, together with a carrier wave $A_c\cos(\omega_c t)$, producing the DSB-SC modulated signal s(t).
The modulated signal is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector, determine the spectrum of the detector output when :

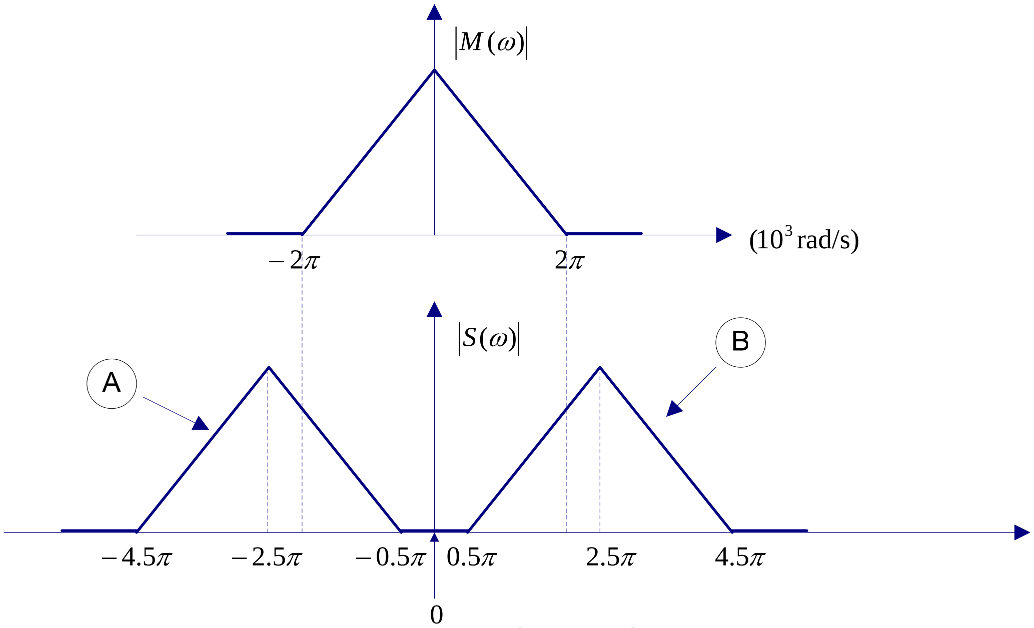
(a) The carrier frequency $\omega_c = 2.5\pi \times 10^3$ rad/sec.
(b) The carrier frequency $\omega_c = 1.5\pi \times 10^3$ rad/sec.

What is the lowest carrier frequency for which each component of the modulated signal s(t) is uniquely determined by m(t) ?



Suggested Answer:

(a) Message bandwidth $\omega_m = 2\pi \times 10^3$ rad/sec and carrier frequency $\omega_c = 2.5\pi \times 10^3$ rad/sec, the spectrum of DSB-SC modulated signal may be depicted as follows:

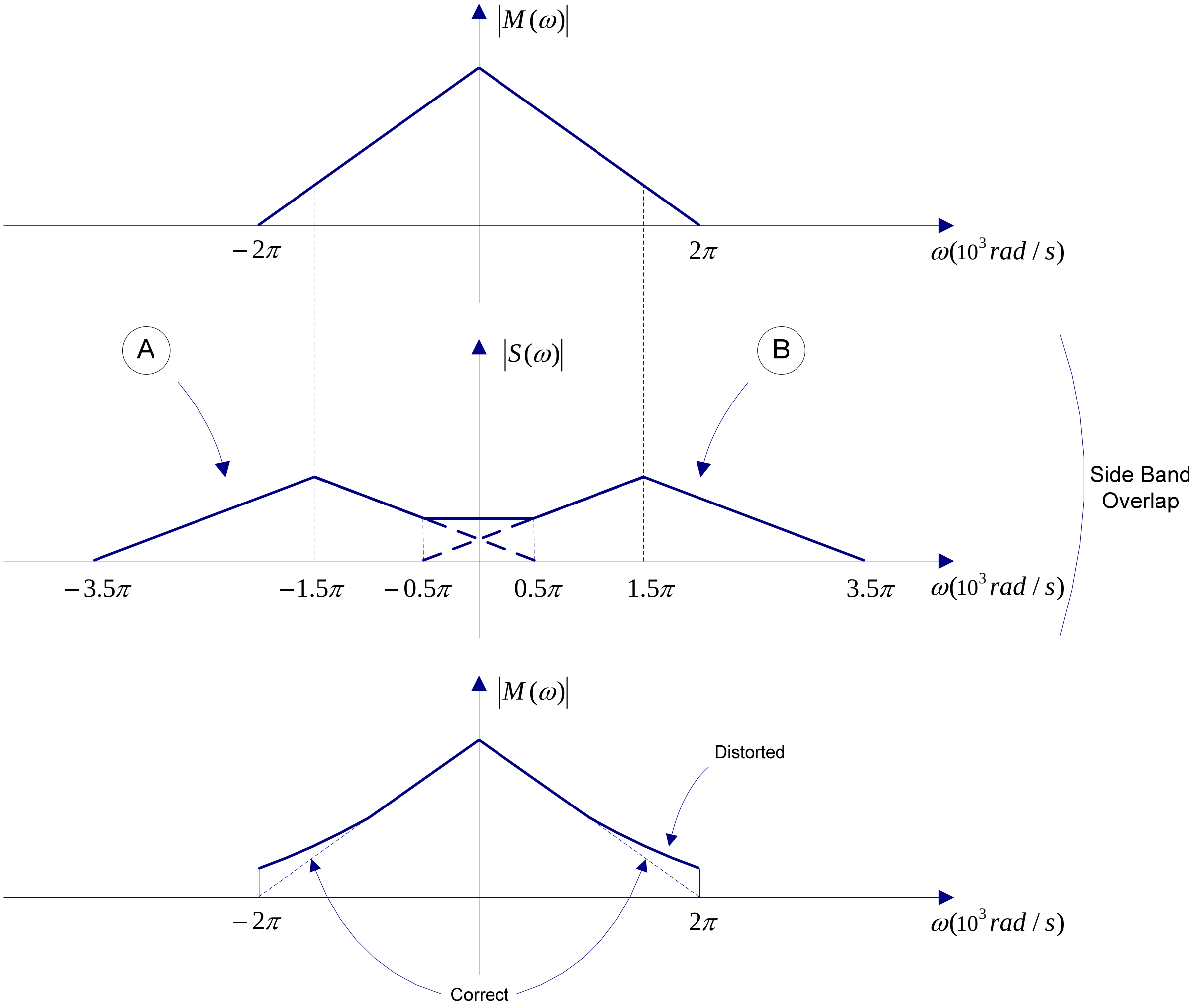


It is clear that there is enough separation between Ⓐ and Ⓑ.

Consequently, when the DSB-SC modulated signal is applied to a coherent detector, the resulting demodulated signal is a replica of the original message signal except for a scale change.

(b) When the carrier frequency is $1.5\pi \times 10^3$ rad/sec,

Ⓐ and Ⓑ overlap.

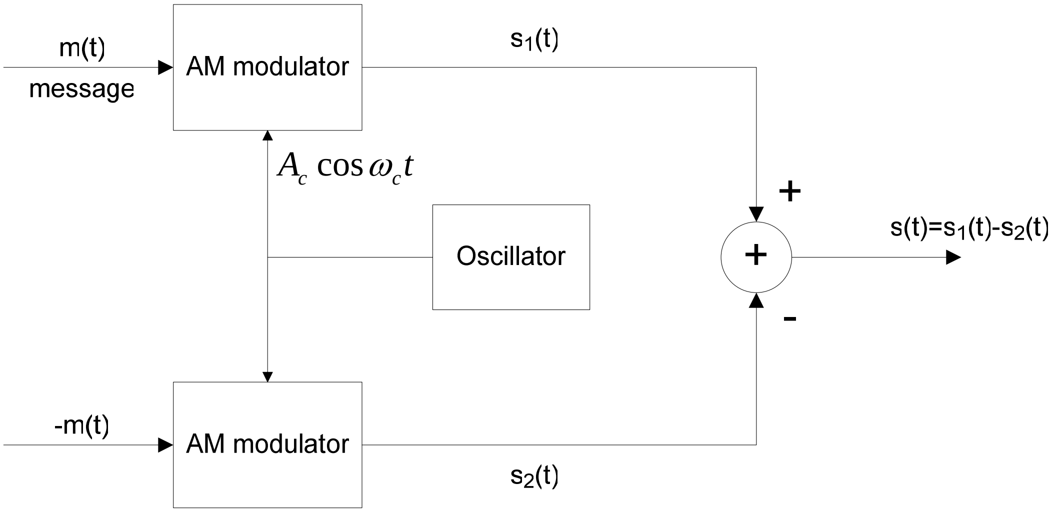


The spectrum $V(\omega)$ refers to demodulated signal appearing at the output of the detector. There is a message distortion. To avoid message distortion in demodulation due to sideband overlap, we must choose the carrier frequency in accordance with the condition $\omega_c \geq \omega_m$. The minimum acceptable value of ω_c is therefore ω_m.

For $\omega_c < \omega_m$, we have sideband overlap and therefore message distortion.

Example: Generation of DSB-SC

DSB-SC is obtained in practice using two AM modulators arranged in a balanced configuration to cancel out the carrier. Figure below shows a diagram of a balanced modulator. The input applied to the top AM modulator is m(t), whereas that applied to the lower AM modulator is –m(t); these two modulators have the same amplitude sensitivity, show that the output s(t) of the balanced modulator consists of a DSB-SC modulated signal.



Suggested Answer:

AM output

$$s_1(t) = A_c(1+k_a m(t))\cos\omega_c t$$

$$s_2(t) = A_c(1-k_a m(t))\cos\omega_c t$$

k_a is same for both modulators (assume)

Amplitude modulation sensetivity

$$\therefore \quad s(t) = s_1(t) - s_2(t) = A_c(1+k_a m(t))\cos\omega_c t - A_c(1-k_a m(t))\cos\omega_c t$$

$$\Rightarrow s(t) = 2k_a A_c m(t)\cos(\omega_c t)$$

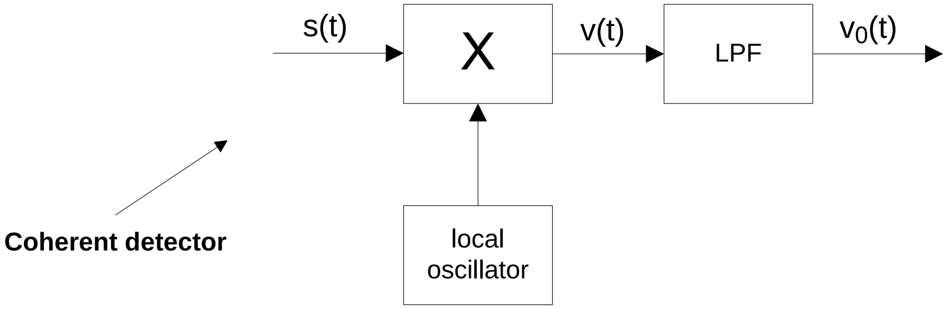
Which is the formula for a DSB-SC modulated signal. For this system to work satisfactorily, the two AM modulators must be carefully matched.

Example:

A DSB-SC modulated signal is demodulated by applying it to a coherent detector. Evaluate the effect of a frequency error $\Delta\omega$ in the local carrier frequency of the detector, measured with respect to the carrier frequency of the incoming DSB-SC signal.

The DSB-SC modulated signal is $s(t) = A_c m(t)\cos\omega_c t$.

Let the local oscillator output in the coherent detector be $\cos(\omega_c t + \underbrace{\Delta\omega t}_{\text{frequency error}})$



$$v(t) = \underbrace{\{A_c m(t)\cos\omega_c t\}}_{s(t)} \times \cos(\omega_c t + \Delta\omega t) = \frac{A_c m(t)}{2}\cos\Delta\omega t + \frac{A_c m(t)}{2}\cos(2\omega_c t + \Delta\omega t)$$

Low-pass filtering v(t) results in the output $v_0(t) = \underbrace{\boxed{\frac{A_c}{2}m(t)}}_{\text{desired component}} \times \cos(\Delta\omega t)$

We have $\overset{\text{DSB-SC}}{s(t) = A_c m(t)\cos\omega_c t}$ Ⓐ

$v_0(t) = \frac{A_c}{2}m(t) \times \cos(\Delta\omega t)$ Ⓑ

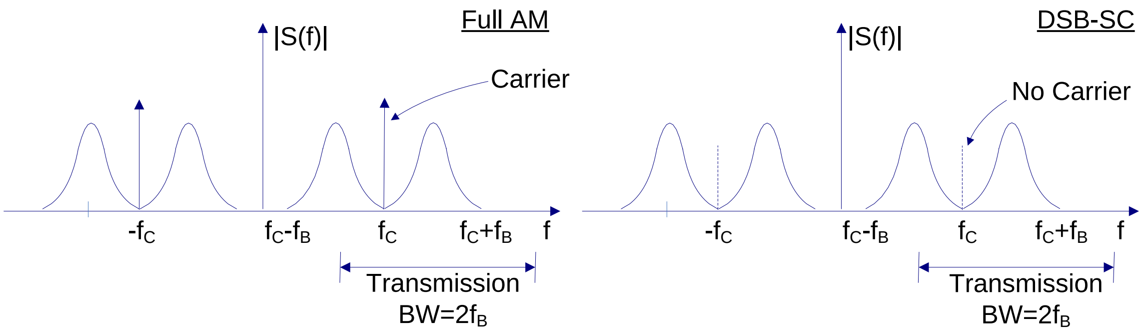
If $\Delta\omega = 0$ then $v_0(t) = \frac{A_c}{2}m(t)$.

The effect of frequency error $\Delta\omega$ in the local oscillator is to produce a new DSB-SC modulated signal with an effective carrier frequency of $\Delta\omega$ (see equations Ⓐ & Ⓑ above).

The coherent detector works properly only when $\Delta\omega = 0$.

3. Single Sideband Amplitude Modulation (SSB)

The full AM (DSB AM) and DSB-SC forms of modulation are wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth (see below).



* Suppressing the carrier reduces the transmission power

In either case, one-half the transmission bandwidth is occupied by the upper sideband of the modulated wave, whereas the other half is occupied by the lower sideband.
The upper and lower sidebands are uniquely related to each other by virtue of their symmetry about the carrier frequency (see fig above).
Given the amplitude and phase spectra of either sideband, we can uniquely determine the other.
Therefore as far as the transmission of information is concerned, only one sideband is necessary, and if both the carrier and the other sideband are suppressed at the transmitter, no information is lost. In this way the channel needs to provide only the message bandwidth.

i.e. **Transmission BW = Message BW (f_B)**, (not $2f_B$)

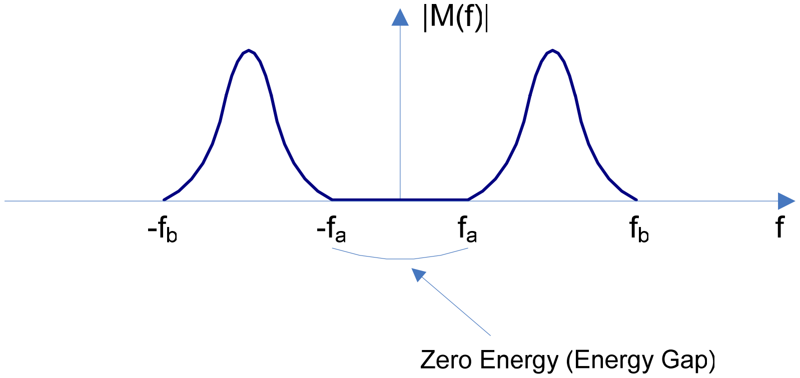
note reduction in transmission BW

When only one sideband is transmitted, the modulation is referred to as **single sideband** (SSB) **modulation.**

Frequency-Domain Description of SSB Modulation:

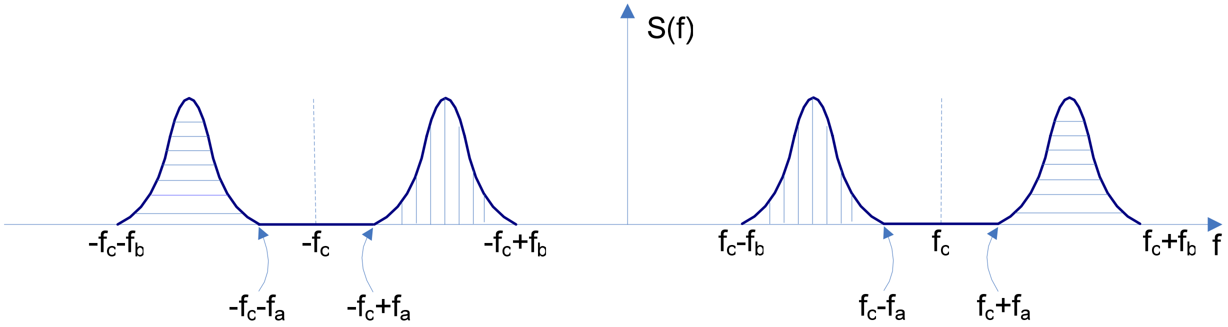
The frequency-domain description of a SSB modulated wave depends on which sideband is transmitted. Consider a message signal m(t) with a spectrum M(f) limited to the band $f_a \le |f| \le f_b$ as shown in Fig 15(a).

Fig 15 (a)



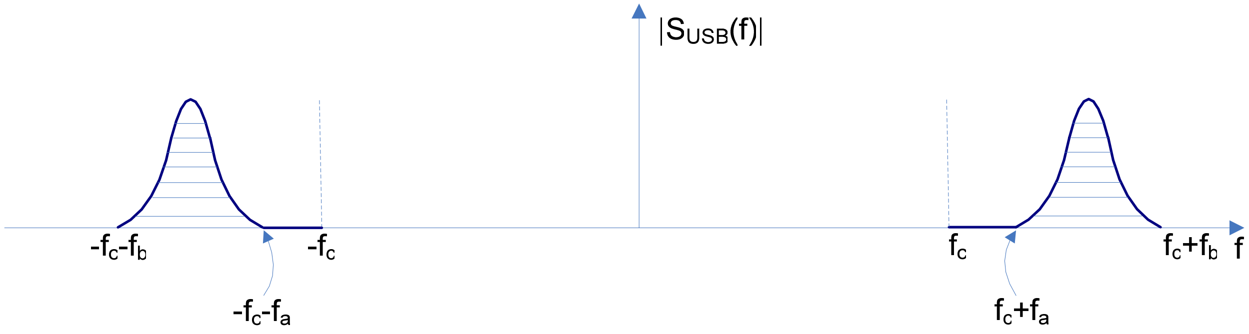
The spectrum of DSB-SC modulated wave, obtained by multiplying m(t) by carrier wave $A_c \cos\omega_c t$ is shown in Fig 15(b).

Fig 15 (b)



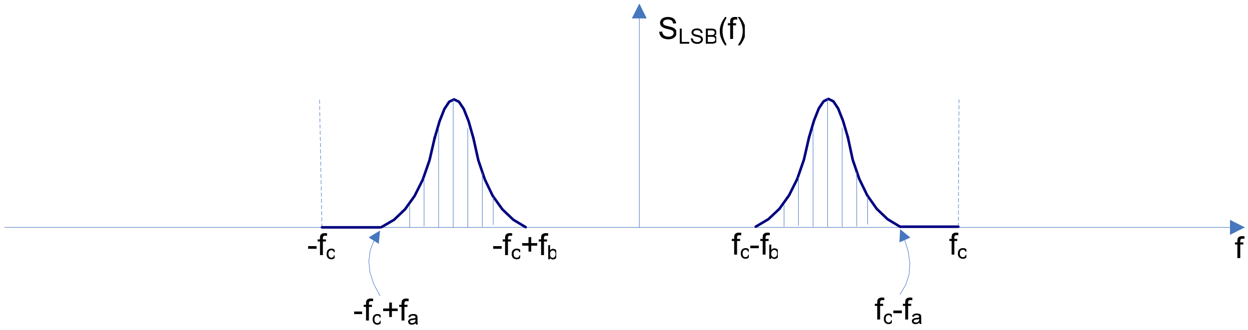
When upper sideband is transmitted, the resulting SSB modulated wave has the spectrum shown in Fig 15(c).

Fig 15 (b)



Magnitude spectrum of SSB modulated wave containing lower sideband only is shown in Fig15(d).

Fig 15 (c)



Thus the essential function of SSB modulation is to translate the spectrum of the modulating wave (message) to a new location in the frequency domain.

Using the frequency domain description in Fig 15, we may develop a scheme for producing SSB modulation (see Fig 16). The scheme consists of a product modulator followed by a band-pass filter.

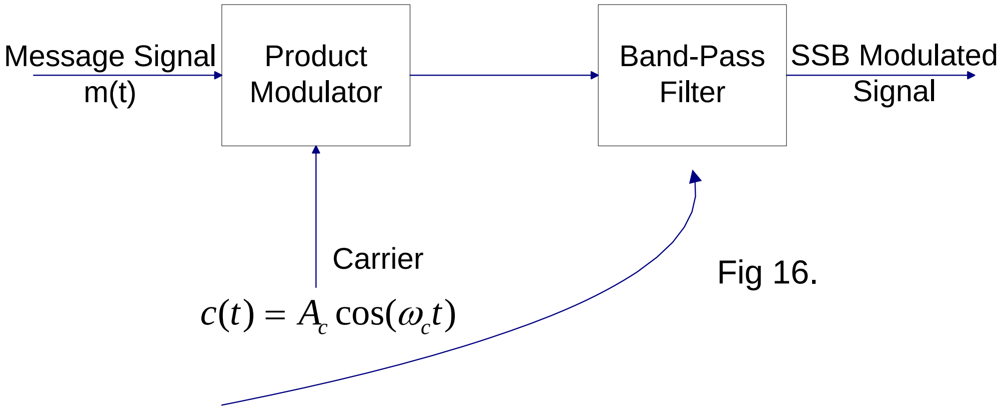


Fig 16.

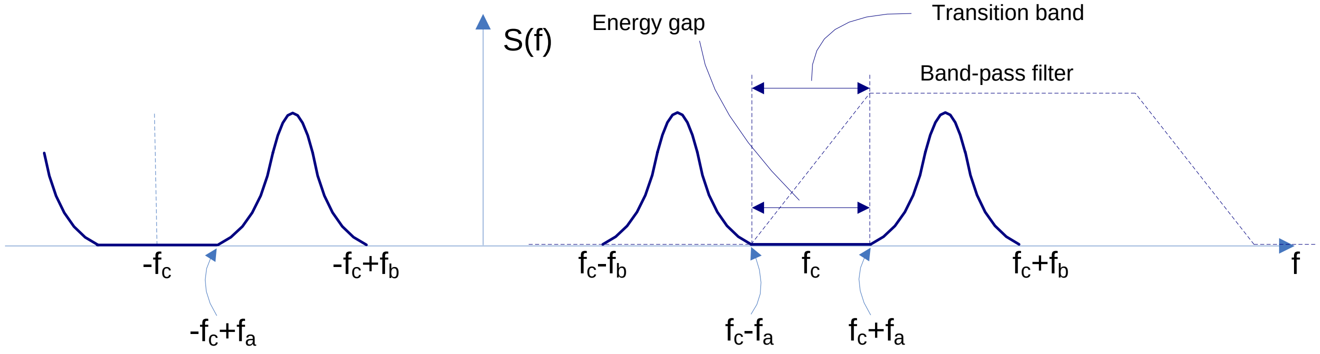
The filter is designed to pass the sideband selected for transmission and suppress the remaining sideband.

For a filter to be physically realizable, the transmission band separating the pass-band from the stop band must have a finite width.

The band-pass filter demands that there be an adequate separation between the lower sideband and the upper sideband of the DSB-SC modulated wave produced at the output of the product modulator.

Such a requirement can only be satisfied if the message signal m(t) applied to the product modulator has an energy gap in its spectrum as shown in Fig 15(c).

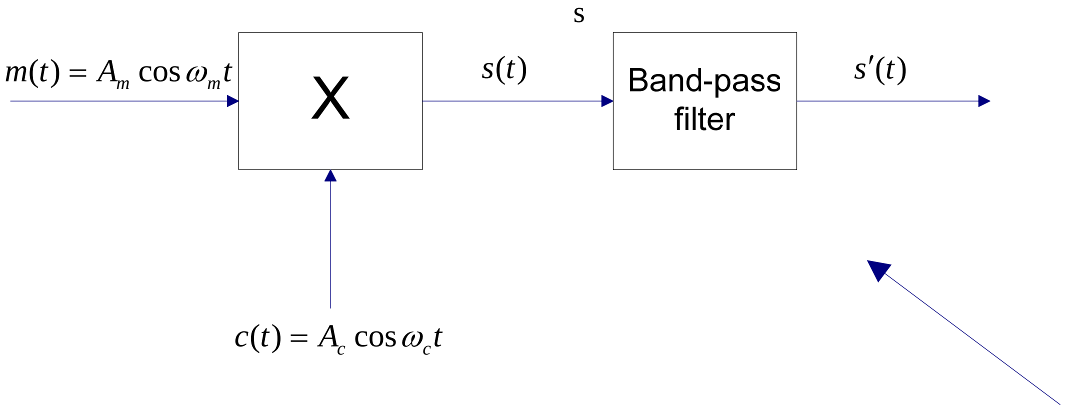
Fig 15 (c)



Fortunately, speech signals for telephone communication do exhibit an energy gap extending from 300Hz to 3000Hz (Telephone speech BW: 300 – 3400 Hz). It is this feature of speech signal that makes SSB modulation well suited for its transmission.
In summary, the transmission bandwidth requirement of a SSB modulator system is one-half that of a standard AM or DBS-SC modulation system.
The benefit of using SSB modulation is therefore derived principally from the reduced bandwidth requirement and the elimination of the high-power carrier wave.
The principal disadvantage of SSB modulation is the cost and complexity of implementing both the transmitter and the receiver.
We have a tradeoff between increased system complexity and improved system performance.

Example:

A SSB modulated wave s(t) is generated using a carrier of frequency ω_c and a sinusoidal modulating wave of frequency ω_m. The carrier amplitude is A_c and that of the modulating wave is A_m. Draw a block diagram of a system for generating SSB modulated wave $s'(t)$. Define $s'(t)$ assuming that (a) only the upper side-frequency is transmitted. (b) the lower side-frequency is transmitted.



Filter method of generating SSB-AM

$$s(t) = A_m A_c \cos \omega_m t . \cos \omega_c t$$

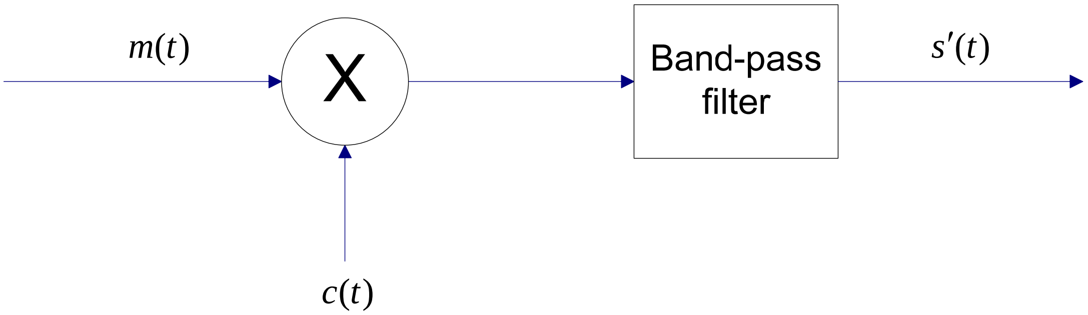
$$s(t) = \frac{A_m A_c}{2} \underset{\substack{\uparrow \\ \text{upper side} \\ \text{frequency}}}{\cos(\omega_c + \omega_m)t} + \frac{A_m A_c}{2} \underset{\substack{\uparrow \\ \text{lower side} \\ \text{frequency}}}{\cos(\omega_c - \omega_m)t}$$

$$s'(t)_{upper} = \frac{A_m A_c}{2} \cos(\omega_c + \omega_m)t$$

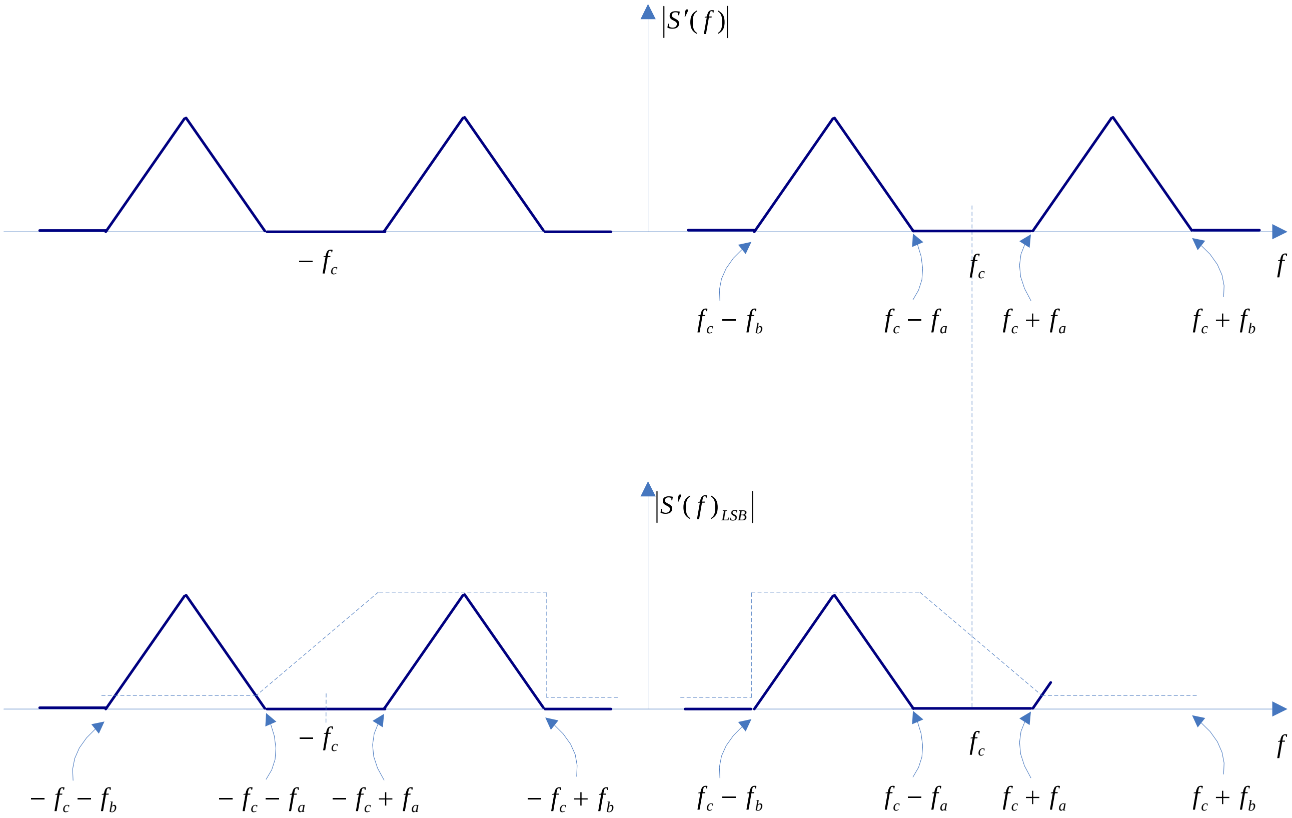
$$s'(t)_{lower} = \frac{A_m A_c}{2} \cos(\omega_c - \omega_m)t$$

Example:

The spectrum of a speech signal lies inside the band $\omega_a \leq |\omega_m| \leq \omega_b$. The carrier frequency is ω_c . Specify the pass band, transmission band and stop band of the band-pass filter (see diagram below) so as to transmit (a) the lower sideband and (b) the upper sideband.



Answer:



To retain lower sideband

(a)

$$Pass-band: f_c - f_b \le |f| \le f_c - f_a$$

$$Transition-band: f_c - f_a \le |f| \le f_c + f_a$$

$$Stop-band: f_c + f_a \le |f| \le f_c + f_b$$

(b)

$$Pass-band: f_c + f_a \le |f| \le f_c + f_b$$

$$Transition-band: f_c - f_a \le |f| \le f_c + f_a$$

$$Stop-band: f_c - f_b \le |f| \le f_c - f_a$$

Time domain Description of SSB modulation

Unlike DSB-SC modulation, the time-domain description of SSB modulation is not as straightforward.
To develop the time-domain description of SSB modulation, we need a mathematical tool known as *Hilbert Transform*.

The Hilbert Transform may be viewed as a **linear filter** with impulse response $h(t) = \dfrac{1}{\pi t}$ and frequency response

$$\boxed{H(f) = \begin{matrix} -j & f > 0 \\ 0 & f = 0 \\ j & f < 0 \end{matrix}} \quad \Longrightarrow \quad H(f) = -jSgn(f)$$

Magnitude →

$$\begin{matrix} |H(f)| = 1 & f > 0 \\ |H(f)| = 1 & f < 0 \end{matrix}$$

Phase →

$$\begin{matrix} \angle H(f) = -90^o & f > 0 \\ \angle H(f) = +90^o & f < 0 \end{matrix}$$

- Magnitude response is unity for all frequencies, both positive and negative.
- The phase is -90^o for positive frequencies $+90^o$ for negative frequencies.

Hilbert Transform is a **phase filter:** it is equivalent to a $-\dfrac{\pi}{2}$ phase shift for positive frequencies and $+\dfrac{\pi}{2}$ phase shift for negative frequencies.
The SSB-AM may be generated by using the following configuration:
(known as phase shift method for SSB generation).

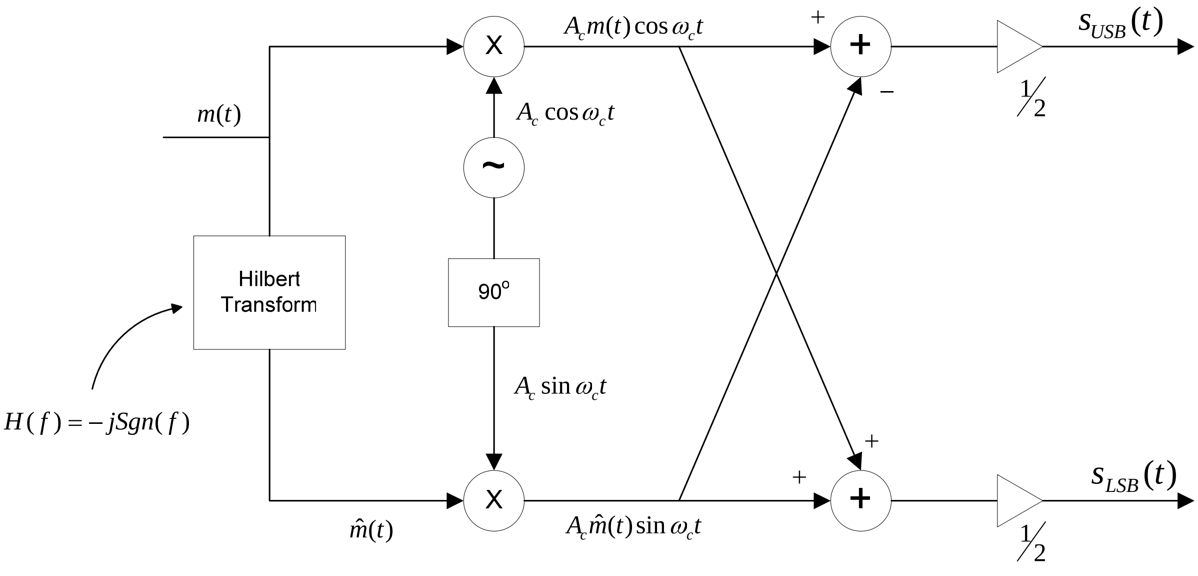


Fig 16.

Suppose the modulating signal m(t),

$$
\begin{aligned}
&m(t) = A_m \cos 2\pi f_m t \qquad f_c >> f_m \\
&then \quad \hat{m}(t) = A_m \cos(2\pi f_m t - 90°) \\
&\hat{m}(t) = A_m \sin 2\pi f_m t
\end{aligned}
$$

If we take upper branch we get the upper side-frequency.

$$\therefore\ s_{USB}(t) = \frac{1}{2}[\underbrace{A_m \cos\omega_m t}_{= m(t)}.A_c \cos\omega_c t - A_m \sin\omega_m t.A_c \sin\omega_c t]$$

$$= \frac{A_m A_c}{2}[\cos\omega_c t.\cos\omega_m t - \sin\omega_c t.\sin\omega_m t]$$

$$s_{USB}(t) = \frac{A_m A_c}{2}\cos(\omega_c + \omega_m)t \quad \longleftarrow \quad USB$$

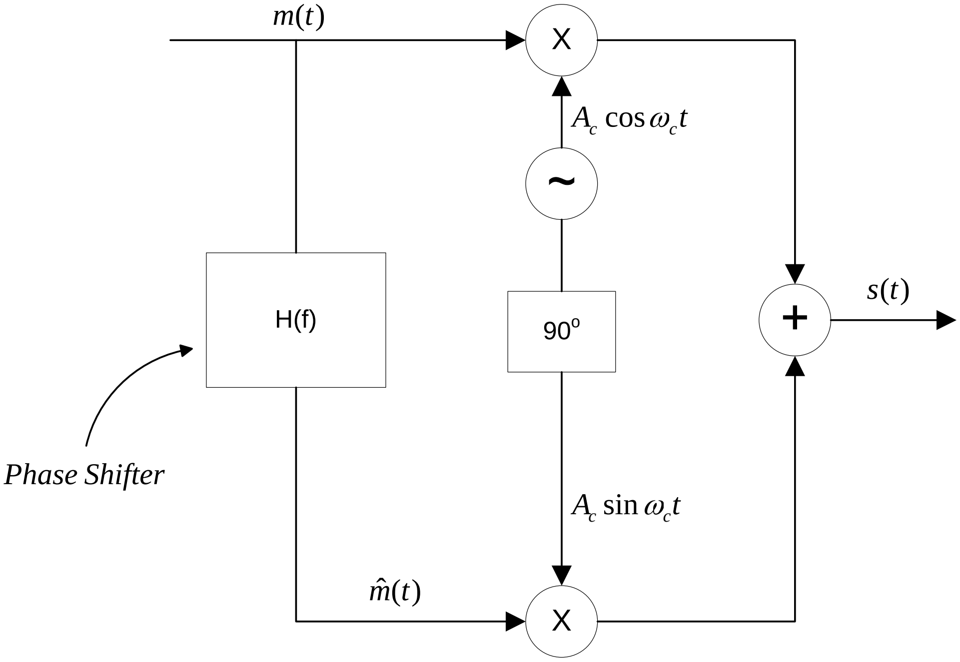
On the other hand, if we take the lower branch,

$$s_{LSB}(t) = \frac{1}{2}[\underbrace{A_m \sin\omega_m t}_{=\hat{m}(t)}.A_c \sin\omega_c t + A_m \cos\omega_m t.A_c \cos\omega_c t]$$

$$= \frac{A_m A_c}{2}[\cos\omega_c t.\cos\omega_m t + \sin\omega_c t.\sin\omega_m t]$$

$$s_{LSB}(t) = \frac{A_m A_c}{2}\cos(\omega_c - \omega_m)t \quad \longleftarrow \quad LSB$$

We can redraw the diagram as



Phase-shift method

$$\boxed{s(t) = A_c m(t)\cos\omega_c t \mp A_c \hat{m}(t)\sin\omega_c t} \qquad (32).$$

$\hat{m}(t)$ is the Hilbert Transform of $m(t)$ and the $\mp$ sign determines which sideband we obtain.

Negative: upper sideband

Positive: lower sideband

Example: **Weave's SSB modulator.**

Consider the modulator of Fig 17.

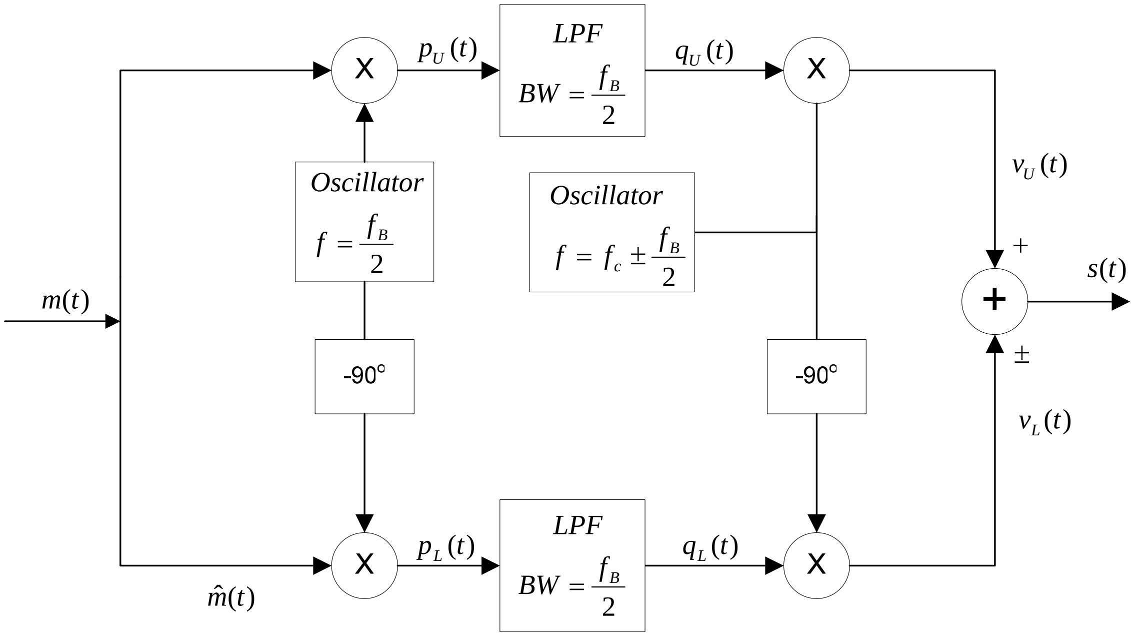


Fig 17.

$$m(t) = \cos\omega_m t \longleftarrow f_m < f_B$$

modulating frequency

oscillator frequency

Show that,

$(a) \quad v_U(t) + v_L(t) = \frac{1}{2}\cos(\omega_c + \omega_m)t$

$(b) \quad v_U(t) - v_L(t) = \frac{1}{2}\cos(\omega_c - \omega_m)t$

$$p_U(t) = \cos\omega_m t.\cos\frac{\omega_B}{2}t$$

$$p_U(t) = \frac{1}{2}\cos(\frac{\omega_B}{2} + \omega_m)t + \frac{1}{2}\cos(\frac{\omega_B}{2} - \omega_m)t$$

Removed by the upper branch LPF

$$q_U(t) = \frac{1}{2}\cos(\frac{\omega_B}{2} - \omega_m)t$$

$$v_U(t) = \frac{1}{2}\cos(\frac{\omega_B}{2} - \omega_m)t.\cos(\pm\frac{\omega_B}{2} + \omega_c)t$$

$$= \frac{1}{4}[\cos(\pm\frac{\omega_B}{2} + \omega_c - \frac{\omega_B}{2} + \omega_m)t + \cos(\pm\frac{\omega_B}{2} - \omega_m + \frac{\omega_B}{2} + \omega_c)t] \qquad (33).$$

The input to the lower LPF is $\cos 2\pi f_m t.\sin 2\pi \frac{f_B}{2} t$.

$$p_L(t) = \frac{1}{2}[\sin(\frac{\omega_B}{2} + \omega_m)t + \sin(\frac{\omega_B}{2} - \omega_m)t]$$

$$p_L(t) = \frac{1}{2}\sin(\frac{\omega_B}{2} - \omega_m)t$$

Removed by the upper branch LPF

$$v_L(t) = \frac{1}{2}\sin(\frac{\omega_B}{2} - \omega_m)t.\sin(\pm\frac{\omega_B}{2} + \omega_m)t$$

$$= \frac{1}{4}[\cos(\pm\frac{\omega_B}{2} + \omega_c - \frac{\omega_B}{2} + \omega_m)t - \cos(\pm\frac{\omega_B}{2} - \omega_m + \frac{\omega_B}{2} + \omega_c)t] \qquad (34).$$

Taking the upper signs,

Add (33) & (34),

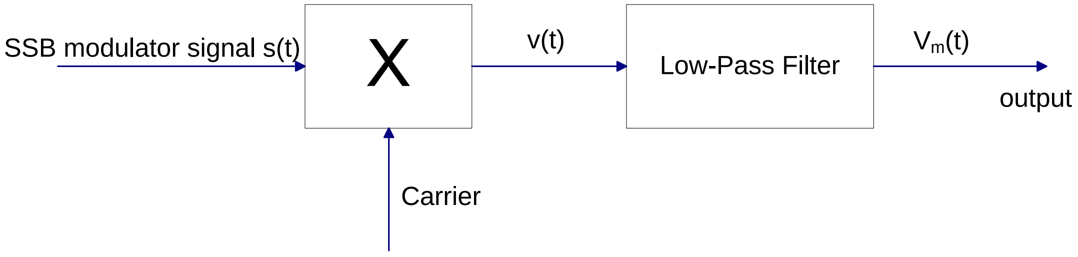
$$s_{USB}(t) = v_U(t) + v_L(t)$$

$$= \frac{1}{4}[\cos(\omega_c + \omega_m)t] + \frac{1}{4}[\cos(\omega_c + \omega_m)t]$$

$$\Rightarrow s_{USB}(t) = \frac{1}{2}[\cos(\omega_c + \omega_m)t]$$ which corresponds to USB.

Demodulation of SSB modulated wave

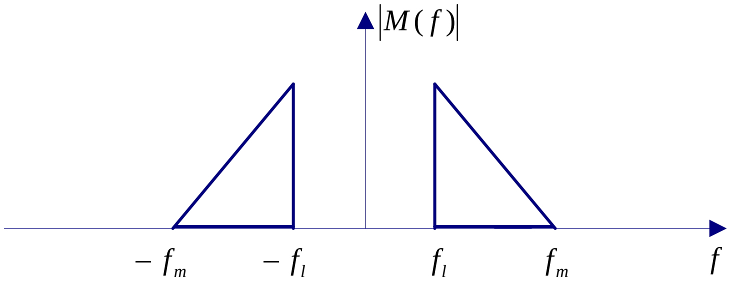
Example: A single sideband modulated wave s(t) is applied to the coherent detector shown below. The cutoff frequency of the low-pass filter is set equal to the highest frequency component of the message signal. Using frequency domain ideas, show that this detector produces an output that is a scaled-version of the original message signal. You may assume that the carrier frequency $\omega_c > \omega_m$.



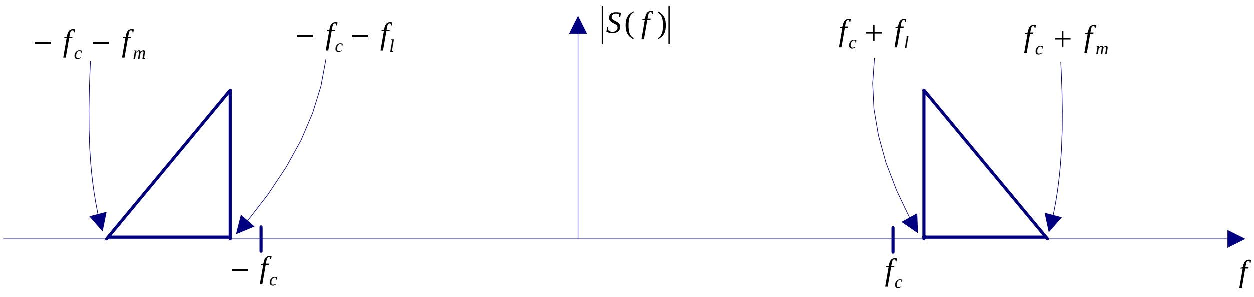
$$c(t) = A_c \cos(\omega_c t)$$

Answer:

Assume that the message signal lies: $f_l \le |f| \le f_m$ and $f_c > f_m$



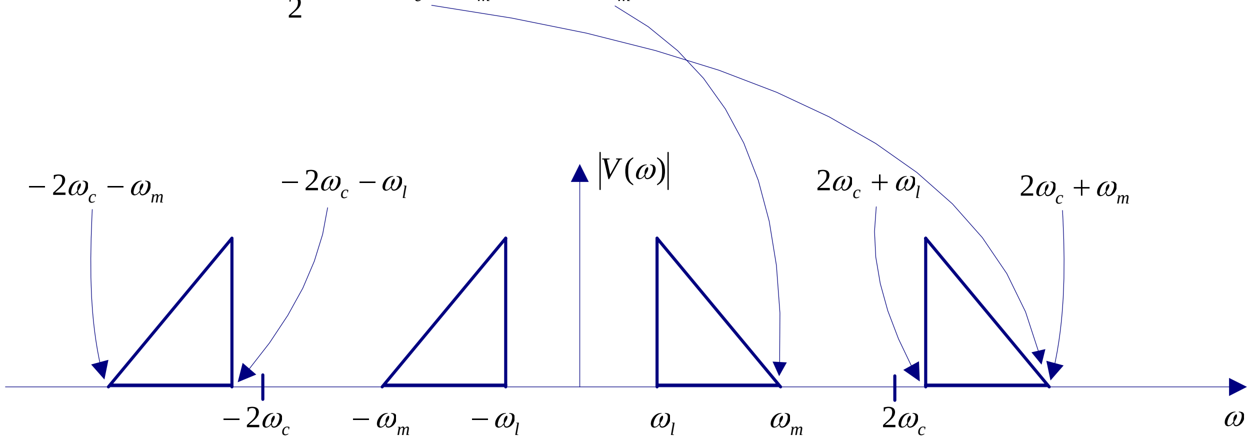
The spectrum of s(t):



Let $s(t) = \frac{1}{2}\cos(\omega_c + \omega_m)t \longleftarrow$ upper side-frequency

$$\therefore\; v(t) = A_c \cos\omega_c t \times \frac{1}{2}\cos(\omega_c + \omega_m)t$$

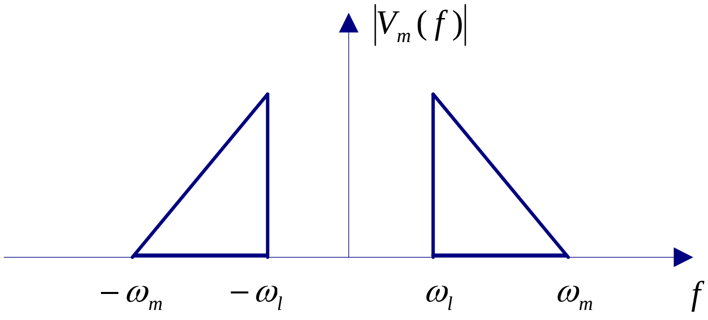
$$= \frac{A_c}{2}[\cos(2\omega_c + \omega_m)t + \cos(\omega_m)t]$$



From the above figure we see that the product modulator output v(t) consists of two components:

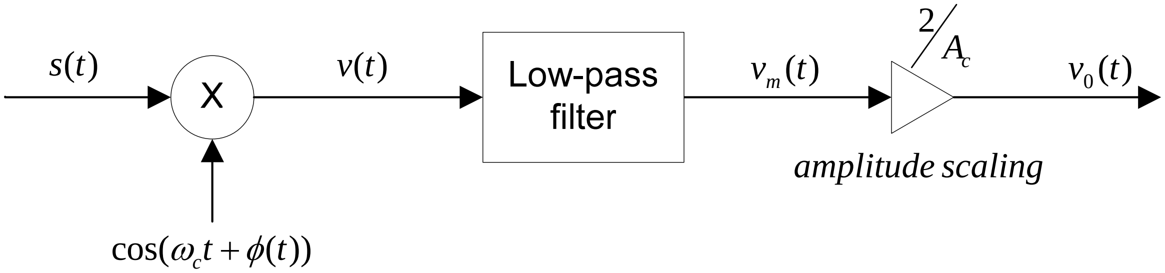
- A scaled version of the original message signal.
- A new SSB modulated signal with **Carrier frequency** $2\omega_c$.

  This latter component is suppressed by the low-pass filter, leaving the original message signal at the output $v_m(t)$.



The simplest technique to demodulate SSB is to multiply s(t) by a demodulation carrier and low-pass filter as explained above(pages …., …..).

We assume a demodulation carrier having a phase error $\phi(t)$.



$$v(t) = s(t).\cos\{\omega_c t + \phi(t)\}$$

$$where \quad s(t) = A_c m(t)\cos\omega_c t \mp A_c \hat{m}(t)\sin\omega_c t \qquad \text{(equation 32).}$$

Let us consider (+) sign

$$v(t) = [A_c m(t)\cos\omega_c t + A_c \hat{m}(t)\sin\omega_c t]\cos[\omega_c t + \phi(t)]$$

$$= A_c m(t)\cos\omega_c t.\cos[\omega_c t + \phi(t)] + A_c \hat{m}(t)\sin\omega_c t.\cos[\omega_c t + \phi(t)]$$

$$= \frac{A_c m(t)}{2}\{\cos[2\omega_c t + \phi(t)] + \cos\phi(t)\} + \frac{A_c \hat{m}(t)}{2}\{\sin[2\omega_c t + \phi(t)] + \sin\phi(t)\}$$

$$\Rightarrow v_m(t) = \frac{A_c m(t)}{2}\cos\phi(t) + \frac{A_c \hat{m}(t)}{2}\sin\phi(t)$$

after low-pass filtering

after amplitude scaling: $v_0(t) = m(t)\cos\phi(t) + \hat{m}(t)\sin\phi(t)$ (35).

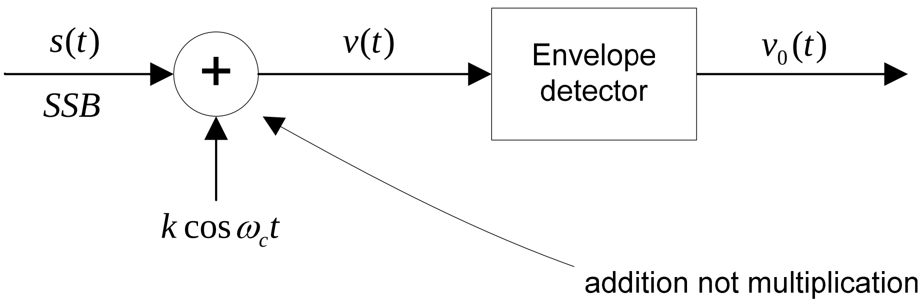
Observation of above equation illustrates that for $\phi(t) = 0$, the demodulated output is the desired message.

$$v_0(t) = m(t)\cos(0) + \hat{m}(t)\sin(0)$$

$$\boxed{v_0(t) = m(t)} \qquad (36).$$

However if, $\phi(t)$ is nonzero, the output consists of the sum of two terms.
The first term is $\{m(t)\cos\phi(t)\}$ a time varying attenuation of the message signal.
The second term is a ‘crosstalk’ term $\{\hat{m}(t)\sin\phi(t)\}$ which can represent serious distortion if $\phi(t)$ is not small.

Another **useful technique** for demodulating an SSB signal is carrier reinsertion. (see below) i.e. add the carrier.



The output of the local oscillator is added to the received signal s(t).

USB

LSB

$$v(t) = s(t) + \cos\omega_c t$$

$$= \underbrace{A_c m(t)\cos\omega_c t \mp A_c\hat{m}(t)\sin\omega_c t}_{= s(t)\quad \text{(page....)}} + k\cos\omega_c t$$

Let us consider negative sign,

$$\therefore\ v(t) = \underbrace{\boxed{[A_c m(t) + k]}\cos\omega_c t - \boxed{A_c\hat{m}(t)}\sin\omega_c t}_{\text{This is the input to the envelope detector}}$$

$$= a(t)\cos\omega_c t - b(t)\sin\omega_c t$$

$$v(t) = R(t)\cos(\omega_c t + \theta) \qquad (37). \qquad \text{where } R(t) = \sqrt{a(t)^2 + b(t)^2} \quad , \quad \theta = \tan^{-1}\frac{b(t)}{a(t)}$$

envelope of an SSB signal, after carrier reinsertion.

$$\therefore \; v_0(t) = \sqrt{(A_c m(t) + k)^2 + A_c \hat{m}(t)} \qquad (38).$$

which is the demodulated output.

If k is chosen large enough such that , $[A_c m(t) + k]^2 >> [A_c \hat{m}(t)]^2$, then the output of the envelope detection becomes: $v_0(t) \approx \sqrt{(A_c m(t) + k)^2}$.

$$\Rightarrow v_0(t) = A_c m(t) + k \qquad (39).$$

From v_0(t) the message signal can easily be extracted.
The development shows that carrier reinsertion requires the locally generated carrier must be phase-coherent with the original modulation carrier.

Note: This requirement is easily accomplished in speech transmission systems. The frequency and phase of the demodulation carrier can be manually adjusted until intelligibility is obtained.