Signals and Systems

- ## Introduction

In communications systems, a signal is usually a function of time. Some examples of signals are human voice, videos etc. Moreover, a communication signal is an electrical voltage $v(t)$ or current $\iota(t)$.

Signals are time-varying quantities such as voltages or current. A system is a combination of devices and networks (subsystems) chosen to perform a desired function .Because of the sophistication of modern communication systems , a great deal of analysis and experimentation with trial subsystems occurs before actual building of the desired system . Thus the communications engineer's tools are mathematical models for signals and systems.

- ## Classification of Signals

1) **Continuous-time and discrete-time signals:** By the term continuous-time signal, we mean a real or complex function of time s (t), where the independent variable *t* is continuous. If *t* is a discrete variable, i.e., s (t) is defined at discrete times, then the signal s (t) is a discrete-time signal. A discrete-time signal is often identified as a sequence of numbers, denoted by {s (n)}, where n is an integer.
2) **Analogue and digital signals**: If a continuous-time signal s (t) can take on any values in a continuous time interval, then s (t) is called an analogue signal. If a discrete-time signal can take on only a finite number of distinct values, {s (n)}, then the signal is called a digital signal.
3) **Deterministic and random signals:** Deterministic signals are those signals whose values are completely specified for any given time. Random signals are those signals that take random values at any given times.
4) **Periodic and nonperiodic signals:** A signal s (t) is a periodic signal if $s(t) = s(t + nT_0)$, where T_0 is called the period and the integer $n > 0$. If $s(t) \neq s(t + T_0)$ for all *t* and any T_0, then s (t) is a nonperiodic or aperiodic signal.
5) **Power and energy signals:** A complex signal s(t) is a power signal if the average normalized power P is finite, where

$$0 < P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} s(t)s^*(t)dt < \infty \qquad 0 < P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 dt < \infty$$

and s*(t) is the complex conjugate of s(t). A complex signal s(t) is an energy signal if the normalized energy E is finite, where

$$0 < E = \int_{-\infty}^{\infty} s(t)s^*(t)\, dt = \int_{-\infty}^{\infty} |s(t)|^2\, dt < \infty$$

In communication systems, the received waveform is usually categorized into the desired part, containing the information signal, and the undesired part, called noise.

Ex.1 – Average power of a sinusoidal signal.
Consider the deterministic signal $x(t) = A\cos(2\pi ft + \theta)$, where A is the amplitude f is the frequency, and θ is the phase.

solution

By definition, the average power is

$$\begin{aligned} P &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cos^2(2\pi ft + \theta)\, dt \\ &= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} [1 + \cos(4\pi ft + 2\theta)]\, dt \\ &= \lim_{T\to\infty} \left(\frac{A^2 T}{2T} + \frac{A^2}{8\pi fT} \Big[\sin(4\pi ft + 2\theta) \Big]_{-T/2}^{T/2} \right) \\ &= \frac{A^2}{2} + \lim_{T\to\infty} \left(\frac{A^2}{8\pi fT} \Big[\sin(4\pi ft + 2\theta) \Big]_{-T/2}^{T/2} \right) \\ &= \frac{A^2}{2} \end{aligned}$$

Note that we have used the identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ in the second line above

- **Some Useful Functions:**

1) **Unit impulse function:** The unit impulse function, also known as the Dirac delta function, δ (t), is defined by :

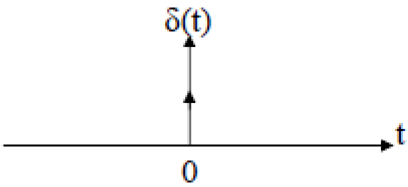
$$\int_{-\infty}^{\infty} s(t)\, \delta(t) dt = s(0)$$

An alternative definition is :

$$\int_{-\infty}^{\infty} \delta\,(t)dt \;=\; 1$$

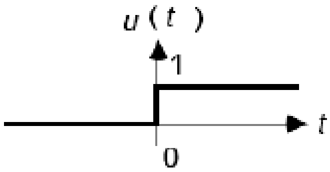
And

$$\delta(t) = \begin{cases} \infty, \ldots\ldots t = 0 \\ 0, \ldots\ldots\ldots t \neq 0 \end{cases}$$



2) Unit step function: The unit step function u(t) is

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



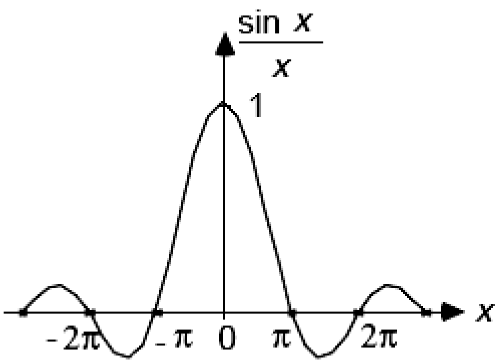
and the unit step function is related to the unit impulse function by :

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\,d\,t \quad \text{and} \quad \frac{du\,(t\,)}{dt} = \delta(t)$$

3) Sampling function :

A sampling function is denoted by :

$$\mathrm{Sa}(x) = \frac{\sin x}{x}$$



4) Sinc function :

A sinc function is denoted by :

$$\mathrm{sinc}\, x = \frac{\sin \pi x}{\pi x} \qquad \text{Hence,} \qquad \mathrm{Sa}(x) = \mathrm{sinc}\left(\frac{x}{\pi}\right)$$

5) Rectangular function :

A single rectangular pulse is denoted by :

$$\Pi\left(\frac{t}{T}\right) = \begin{cases} 1, & |t| \le \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases}$$

6) Triangular function :

$$\Lambda\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \le T \\ 0, & |t| > T \end{cases}$$

- **Bandwidth**

The bandwidth of a communication link, or in general any system, was loosely defined as the width of the frequency interval such that input sinusoidal frequencies within this interval will appear at the output without significant amplitude or phase change.

Bandwidth is related to maximum pulse transmission rate and hence data transmission rate for the link, and is clearly an important characteristic.

*A more precisely defined characteristic incorporating bandwidth information is the frequency response of the link or system.