- We can also write a PSK signal as:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_c t + \frac{2\pi(i-1)}{M} \right)$$

$$= \sqrt{\frac{2E}{T}} \left[\cos \frac{2\pi(i-1)}{M} \cos \omega_c t - \sin \frac{2\pi(i-1)}{M} \sin \cos \omega_c t \right]$$

- Furthermore, $s_1(t)$ may be represented as a linear combination of two orthogonal functions $\psi_1(t)$ and $\psi_2(t)$ as follows

$$s_i(t) = \sqrt{E} \cos \frac{2\pi(i-1)}{M} \psi_1(t) - \sqrt{E} \sin \frac{2\pi(i-1)}{M} \psi_2(t)$$

Where

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos[\omega_c t] \quad and \quad \psi_2(t) = \sqrt{\frac{2}{T}} \sin[\omega_c t]$$

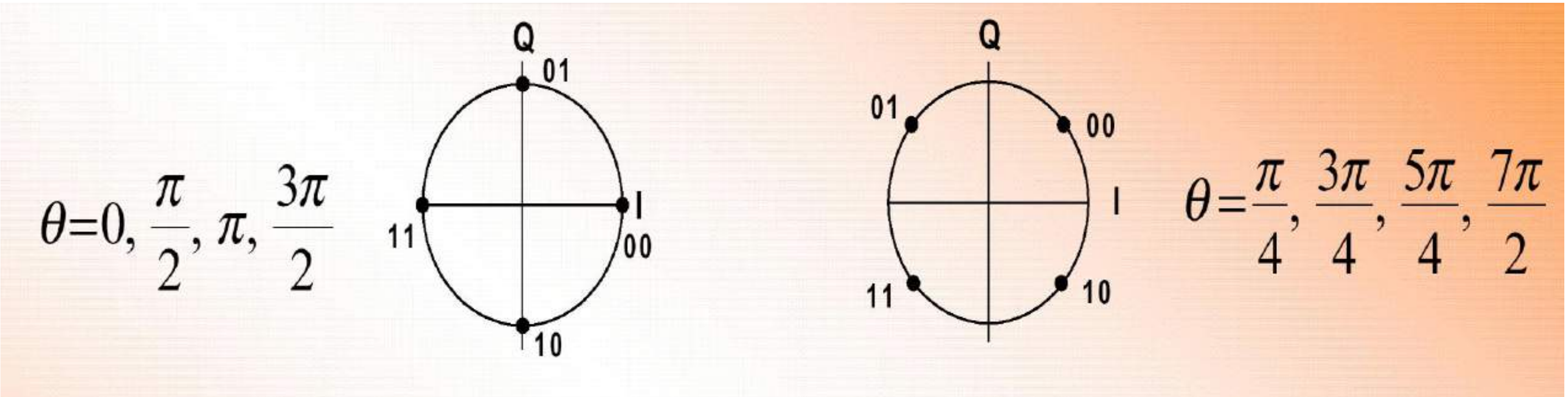
- Using the concept of the orthogonal basis function, we can represent PSK signals as a two dimensional vector

$$s_i(t) = \left(\sqrt{E_b} \cos \frac{2\pi(i-1)}{M} \psi_1, \sqrt{E_b} \sin \frac{2\pi(i-1)}{M} \psi_2 \right)$$

- For M-ary phase modulation ***M* = 2*^k***, where ***k*** is the number of information bits per transmitted symbol

- In an M-ary system, one of ***M* ≥ 2** possible symbols, $s_1(t), \ldots, s_m(t)$, is transmitted during each T_s-second signaling interval

- The mapping or assignment of ***k*** information bits into $M = 2^k$ possible phases may be performed in many ways, e.g. for ***M = 4***



- A preferred assignment is to use "Gray code" in which adjacent phases differ by only one binary digit such that only a single bit error occurs in a ***k***-bit sequence. Will talk about this in detail in the next few slides.

- It is also possible to transmit data encoded as the phase change (phase difference) between consecutive symbols
  - This technique is known as Differential PSK (DPSK)

- There is no non-coherent detection equivalent for PSK except for DPSK

M-ary PSK

- In MPSK, the phase of the carrier takes on one of *M* possible values

$$\phi_i(t) = \frac{2\pi(i-1)}{M}, \quad i = 1,2,\ldots\ldots, M$$

- Thus, MPSK waveform is expressed as

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\omega_0 t + \frac{2\pi(i-1)}{M}\right]$$

$$s_i(t) = g(t)\cos\left[\omega_0 t + \frac{2\pi(i-1)}{M}\right]$$

$M = 2^k$	$MPSK$
2	$BPSK$
4	$QPSK$
8	$8-PSK$
16	$16-PSK$
....	

- Each $s_i(t)$ may be expanded in terms of two basis function $\Psi_1(t)$ and $\Psi_2(t)$ defined as

$$\psi_1(t) = \sqrt{\frac{2}{T_s}} \cos\omega_c t, \qquad \psi_2(t) = \sqrt{\frac{2}{T_s}} \sin\omega_c t,$$

Quadrature PSK (QPSK)

- Two BPSK in phase quadrature
- QPSK (or 4PSK) is a modulation technique that transmits 2-bit of information using 4 states of phases
- For example

2-bit Information	ø
00	0
01	π/2
10	π
11	3π/2

Each symbol corresponds to two bits

- General expression:

$$s_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + \frac{2\pi(i-1)}{M}\right], \quad i = 1,2,3,4 \quad 0 \le t \le T_s$$

- The signals are:

$$s_0 = \sqrt{\frac{2E_s}{T_s}} \cos(\omega_c t) \qquad s_1 = \sqrt{\frac{2E_s}{T_s}} \cos(\omega_c t + \frac{\pi}{2}) = -\sqrt{\frac{2E_s}{T_s}} \sin(\omega_c t)$$

$$s_2 = \sqrt{\frac{2E_s}{T_s}} \cos(\omega_c t + \pi) = -\sqrt{\frac{2E_s}{T_s}} \cos(\omega_c t)$$

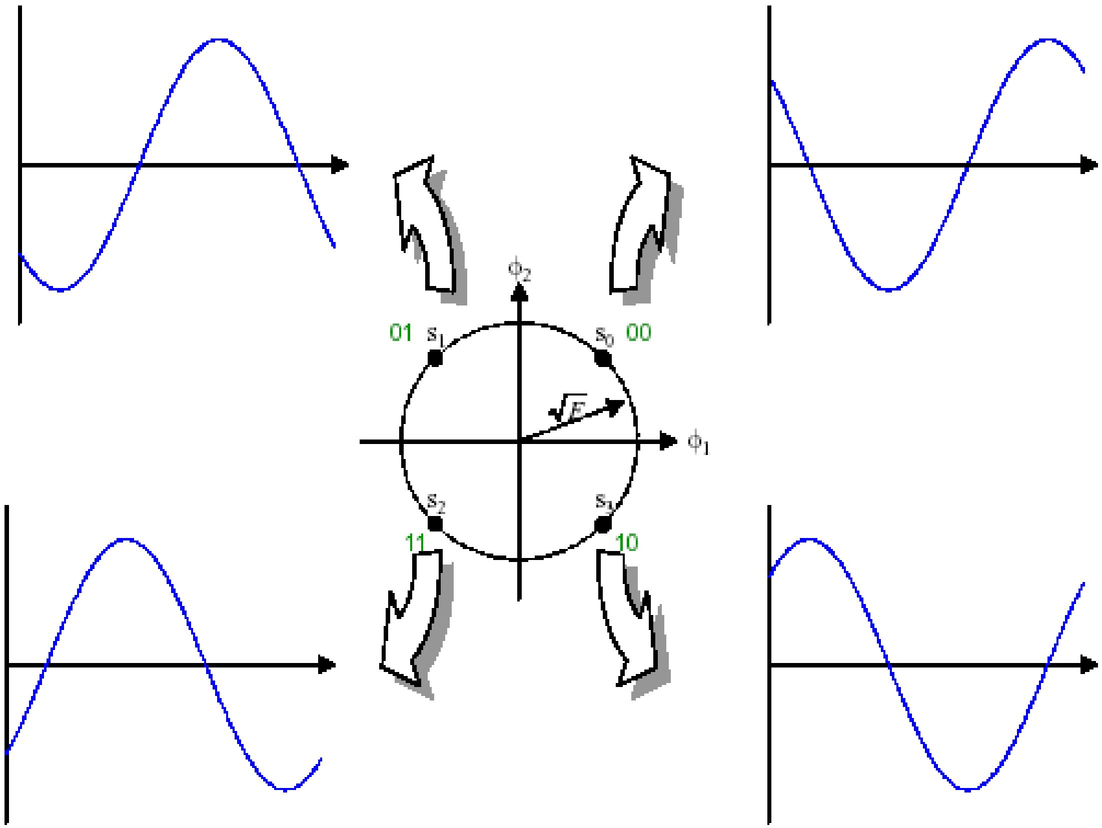
$$s_3 = \sqrt{\frac{2E_s}{T_s}} \cos(\omega_c t + \frac{3\pi}{2}) = \sqrt{\frac{2E_s}{T_s}} \sin(\omega_c t)$$

$$s_{0,2}(t) = \pm\sqrt{\frac{2E_s}{T_s}} \cos \omega_c t, \quad \phi - shift\ of\ 0^0\ and\ 180^0$$

$$s_{1,3}(t) = \pm\sqrt{\frac{2E_s}{T_s}} \sin \omega_c t, \quad \phi - shift\ of\ 90^0\ and\ 270^0$$

- We can also have:

$$s_{QPSK}(t)=\sqrt{\frac{2E_s}{T_s}}\cos\left[\omega_c t+\frac{2\pi(i-1)}{M}-\frac{\pi}{4}\right],\quad i=1,2,3,4\quad 0\le t\le T_s$$



- One of 4 possible waveforms is transmitted during each signaling interval Ts
  - i.e., 2 bits are transmitted per modulation symbol → Ts=$\boldsymbol{2T_b}$)
- In QPSK, both the in-phase and quadrature components are used
- The **I** and **Q** channels are aligned and phase transition occur once every $\boldsymbol{T_s = 2T_b}$ seconds with a maximum transition of 180 degrees
- From

$$s_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[2\pi f_c t + \frac{2\pi(i-1)}{M}\right]$$

- As shown earlier we can use trigonometric identities to show that

$$s_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[\frac{2\pi(i-1)}{M}\right] \cos(\omega_c t) - \sqrt{\frac{2E_s}{T_s}} \sin\left[\frac{2\pi(i-1)}{M}\right] \sin(\omega_c t)$$

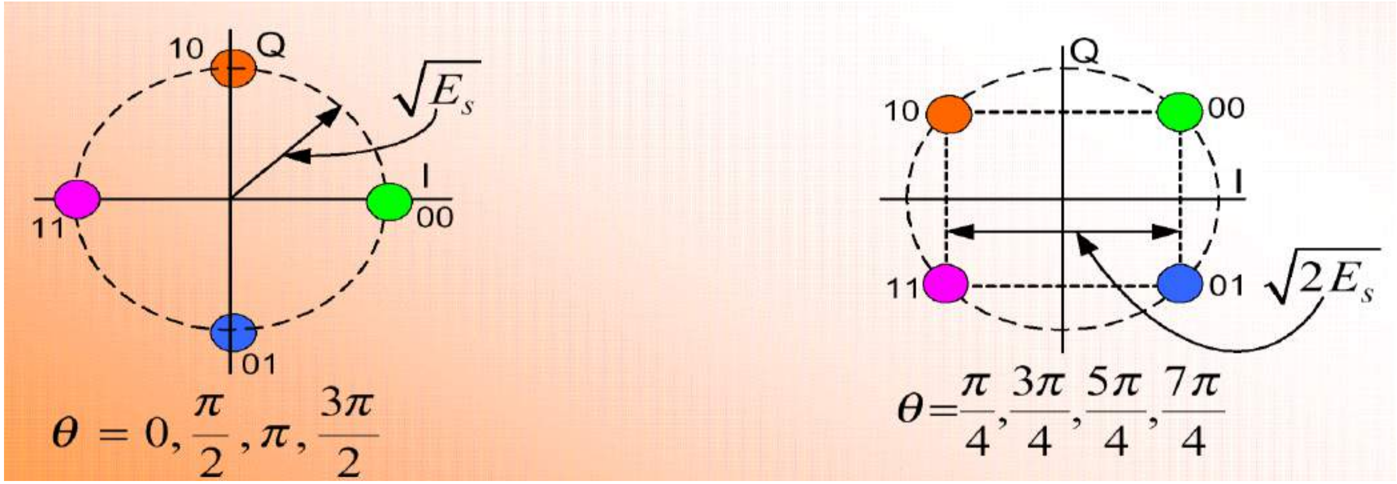
- In terms of basis functions

$$\psi_1(t) = \sqrt{\frac{2}{T_s}} \cos 2\pi f_c t \quad and \quad \psi_2(t) = \sqrt{\frac{2}{T_s}} \sin 2\pi f_c t$$

we can write $s_{QPSK}(t)$ as

$$s_{QPSK}(t) = \left\{ \sqrt{E_s} \cos\left[\frac{2\pi(i-1)}{M}\right]\psi_1(t) - \sqrt{E_s} \sin\left[\frac{2\pi(i-1)}{M}\right]\psi_2(t) \right\}$$

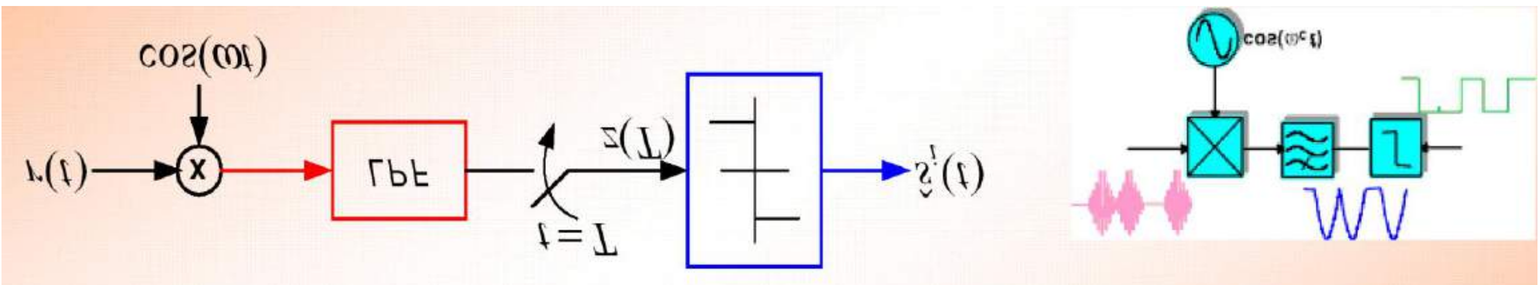
- With this expression, the constellation diagram can easily be drawn
- For example:



Coherent Detection

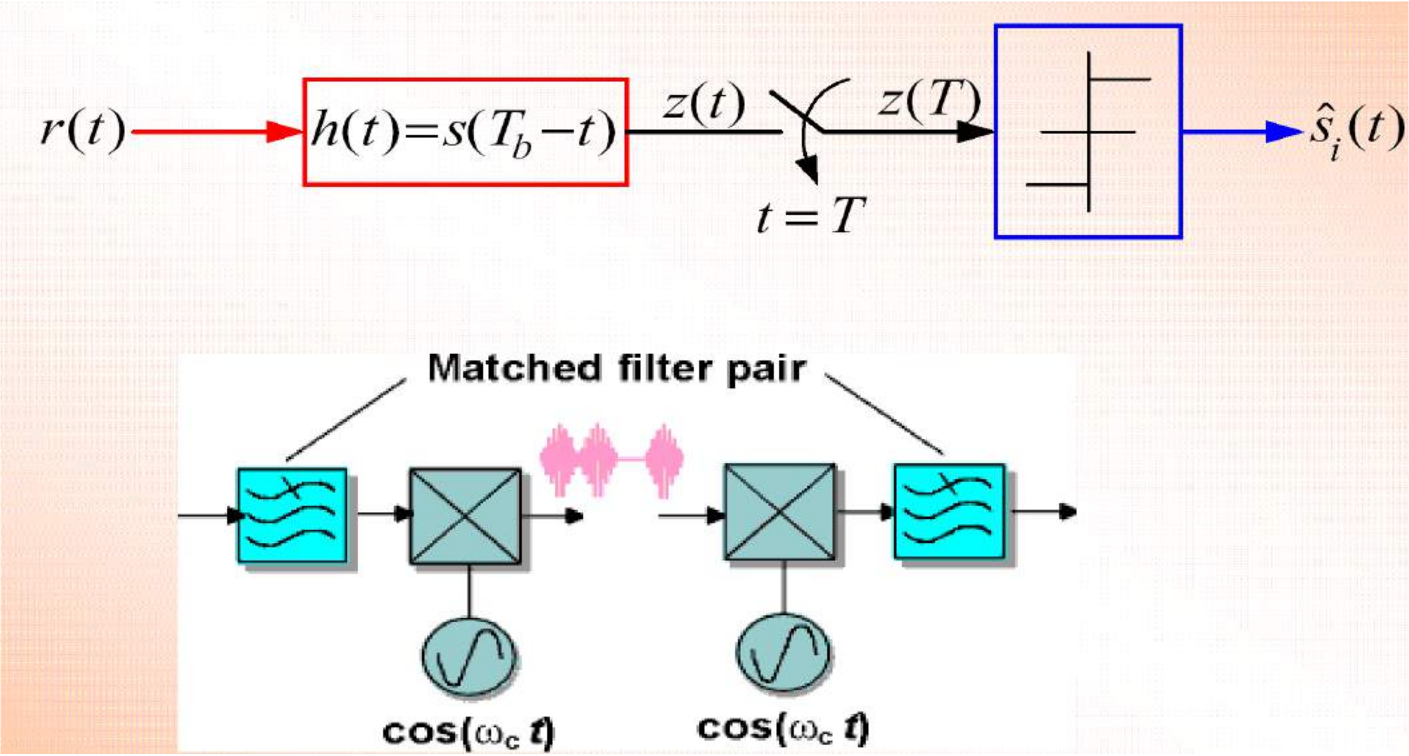
1. Coherent Detection of PSK

- Coherent detection requires the phase information
- A coherent detector operates by mixing the incoming data signal with a locally generated carrier reference and selecting the difference component from the mixer output



- Multiplying *r(t)* by the receiver LO (say $A \cos(\omega_c t)$) yields a signal with a baseband component plus a component at $2f_c$
- The LPF eliminates the high frequency component
- The output of the LPF is sampled once per bit period
- The sampled value *z(T)* is applied to a decision rule
  - *z(T)* is called the **decision statistic**

- Matched filter receiver



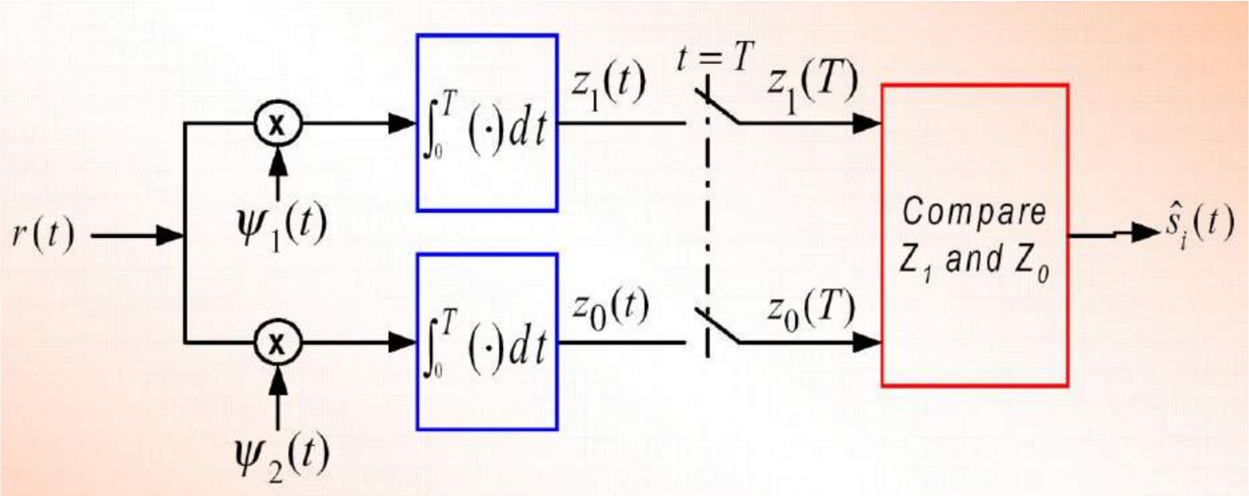
- A MF pair such as the root raised cosine filter can thus be used to shape the source and received baseband symbols
- In fact this is a very common approach in signal detection in most bandpass data modems

2. Coherent Detection of MPSK

- QPSK receiver is composed of 2 BPSK receivers
  - one that locks on to the sine carrier and
  - the other that locks onto the cosine carrier

$$\psi_1(t) = A\cos\omega_0 t$$

$$\psi_2(t) = A\sin\omega_0 t$$



$$z_0(t) = \int_0^{T_s} s_0(t)\psi_1(t)dt = \int_0^{T_s} (A\cos\omega_0 t)\ (A\cos\omega_0 t)dt = \frac{A^2 T_s}{2} \triangleq L_0$$

$$z_1(t) = \int_0^{T_s} s_0(t)\psi_2(t)dt = \int_0^{T_s} (A\cos\omega_0 t)\ (A\sin\omega_0 t)dt = 0$$

Output	$S_0(t)$	$S_1(t)$	$S_2(t)$	$S_3(t)$
Z_0	Lo	0	-Lo	0
Z_1	0	-Lo	0	Lo

$$L_0 = \frac{A^2 T_s}{2} \cos\frac{\pi}{4}$$

- If $\psi_1(t) = A\cos(\omega_0 t + 45^o)$ *and* $\psi_2(t) = A\cos(\omega_0 t - 45^o)$

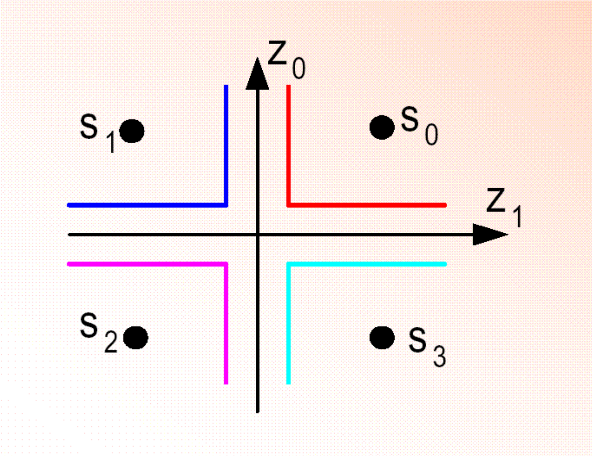
Output	$S_0(t)$	$S_1(t)$	$S_2(t)$	$S_3(t)$
Z_0	Lo	-Lo	-Lo	Lo
Z_1	Lo	Lo	-Lo	-Lo

- Decision:

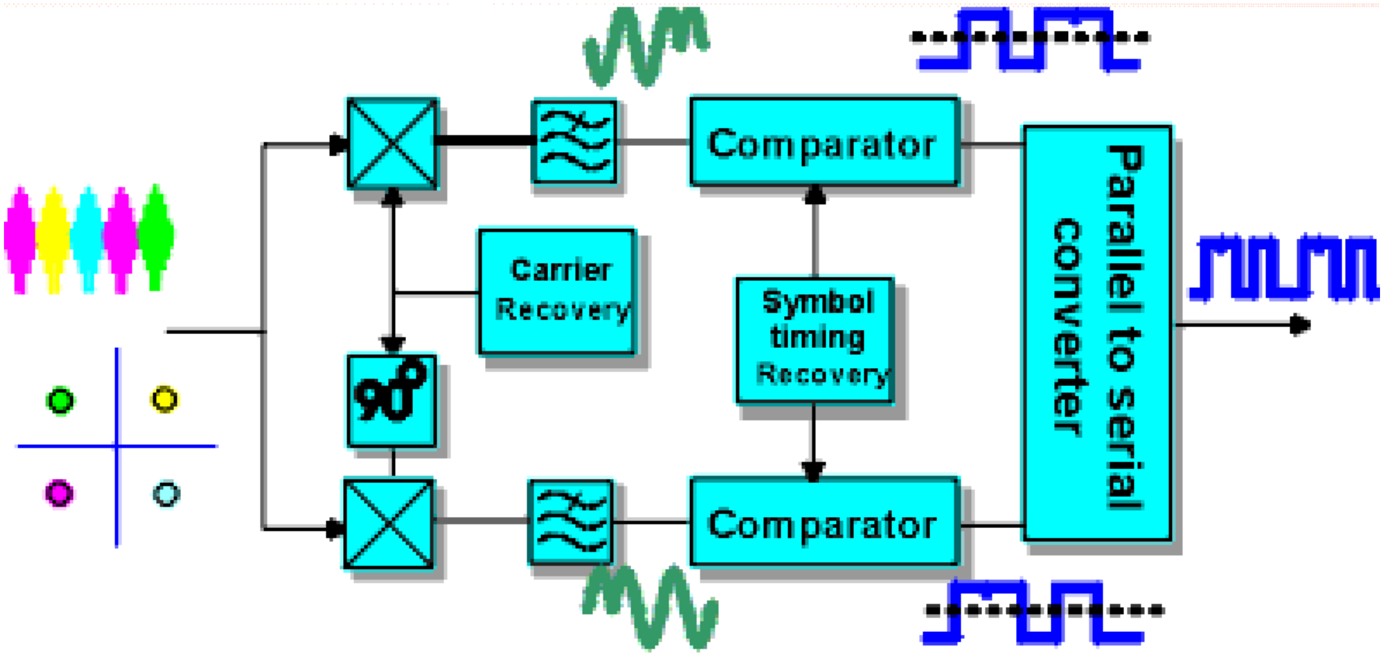
1. Calculate $z_i(t)$ as

$$z_i(t) = \int_0^T r(t)\,\psi_i(t)dt$$

2. Find the quadrant of (Z_0, Z_1)



- A coherent QPSK receiver requires accurate carrier recovery using a 4th power process, to restore the 90º phase states to modulo 2π



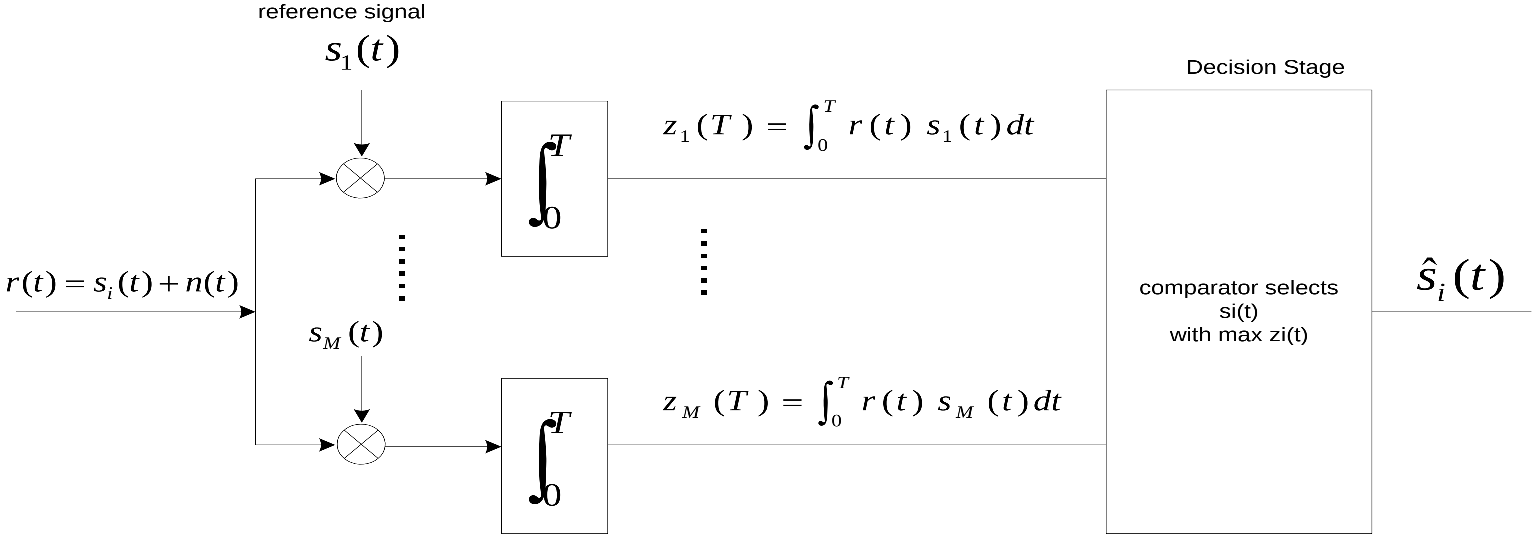
QPSK detection

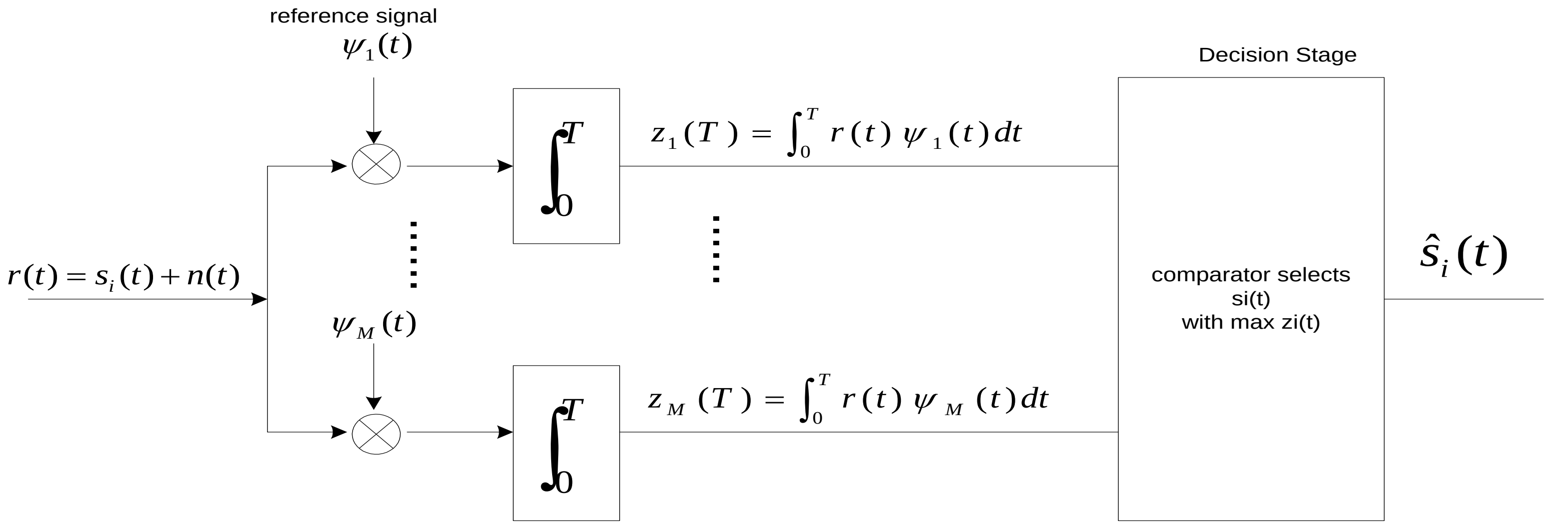
4.3 Detection of Signals in Gaussian Noise

- Detection models at baseband and passband are identical
- Equivalence theorem (for linear systems):
  - Linear signal processing on passband signal and eventual heterodyning to baseband is equivalent to first heterodyning passband signal to baseband followed by linear signal processing
  - Where

    Heterodyning = Process resulting in spectral shift in signal e.g. mixing
- Performance Analysis and description of communication systems is usually done at baseband for simplicity

4.3.2 Correlation Receiver





4.4 Coherent Detection

4.4.1 Coherent Detection of PSK

- Consider the following binary PSK example

$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi) \quad 0 \le t \le T$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi + \pi) \qquad = -\sqrt{\frac{2E}{T}} \cos(\omega_0 t + \phi)$$

n(t) = zero-mean Gaussian random process

- Where φ : phase term is an arbitrary constant

  E: signal energy per symbol

  T: Symbol duration

- Single basis function for this antipodal case:

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t \qquad for \quad 0 \le t \le T$$

- Transmitted signals $s_i(t)$ in terms of $\psi_1(t)$ and coefficients $a_{i1}(t)$ are

$$s_i(t) = a_{i1}(t)\psi_1(t)$$

$$s_1(t) = a_{11}(t)\psi_1(t) = \sqrt{E}\,\psi_1(t)$$

$$s_2(t) = a_{21}(t)\psi_1(t) = -\sqrt{E}\,\psi_1(t)$$

- Assume that s1 was transmitted, then values of product integrators with reference to ψ1 are

$$E\{z_1 \mid s_1\} = E\left\{ \int_0^T \sqrt{E}\,\psi_1^2(t) + n(t)\psi_1(t)dt \right\}$$

$$E\{z_2 \mid s_1\} = E\left\{ \int_0^T -\sqrt{E}\,\psi_1^2(t) + n(t)\psi_1(t)dt \right\}$$

$$E\{z_1 \mid s_1\} = E\left\{\int_0^T \frac{2}{T}\sqrt{E}\cos^2\omega_0 t + n(t)\sqrt{\frac{2}{T}}\cos\omega_0 t\ dt\right\} = \sqrt{E}$$

$$E\{z_2 \mid s_1\} = E\left\{\int_0^T -\frac{2}{T}\sqrt{E}\cos^2\omega_0 t + n(t)\sqrt{\frac{2}{T}}\cos\omega_0 t\ dt\right\} = -\sqrt{E}$$

where E{n(t)}=0

- Decision stage determines the the location of the transmitted signal within the signal space
- For antipodal case choice of $\psi_1(t) = \sqrt{2/T}\ \cos w_0 t$ normalizes $E\{z_i(T)\}$ to $\pm\sqrt{E}$
- Prototype signals $s_i(t)$ are the same as reference signals $\psi_j(t)$ except for normalizing scale factor
- Decision stage chooses signal with largest value of $z_i(T)$

4.4.2 Sampled Matched Filter

- The impulse response h(t) of a filter matched to s(t) is:

$$h(t) = \begin{cases} s(T-t) & 0 \le t \le T \\ 0 & elsewhere \end{cases} \qquad \text{(eq 4.26)}$$

- Let the received signal r(t) comprise a prototype signal $s_i(t)$ plus noise n(t)
- Bandwidth of the signal is W =1/2T where T is symbol time then
- F_s= 2W = 1/T
- Sample at t =kT_s. This allows us to use discrete notation:

$$r(k) = s_i(k) + n(k) \quad i = 1,2 \;\; k = 0,1,\ldots$$

- Let $c_i(n)$ be the coefficients of the MF where n is the time index and N represents the samples per symbol

$$c_i(n) = s_i[(N-1)-n] \qquad \text{(eq 4.27)}$$

- Discrete form of convolution integral suggests

$$z_i(k) = \sum_{n=0}^{N-1} r(k-n)\, c_i(n) \quad K = 0,1,\ldots..,\text{modulo} - \text{N} \qquad \text{(eq 4.28)}$$

- Since noise is assumed to have zero mean, so the expected value of a received sample is:

$$E\{r(k)\} = s_i(k) \qquad i = 1,2$$

- Therefore, if $s_i(t)$ is transmitted, the expected MF output is:

$$E\{z_i(k)\} = \sum_{n=0}^{N-1} s_i(k-n)\, c_i(n) \quad K = 0,1,\ldots..,\text{modulo} - \text{N}$$

(eq 4.29)

- Combining eq (4.27) and (eq 4.29) to express the correlator outputs at time k = N –1 = 3:

$$z_1(k=3) = \sum_{n=0}^{3} s_1(3-n)\, c_1(n) = 2 \qquad \text{(eq 4.30a)}$$

$$z_2(k=3) = \sum_{n=0}^{3} s_1(3-n)\, c_2(n) = -2 \qquad \text{(eq 4.30b)}$$

Sampled Matched Filter

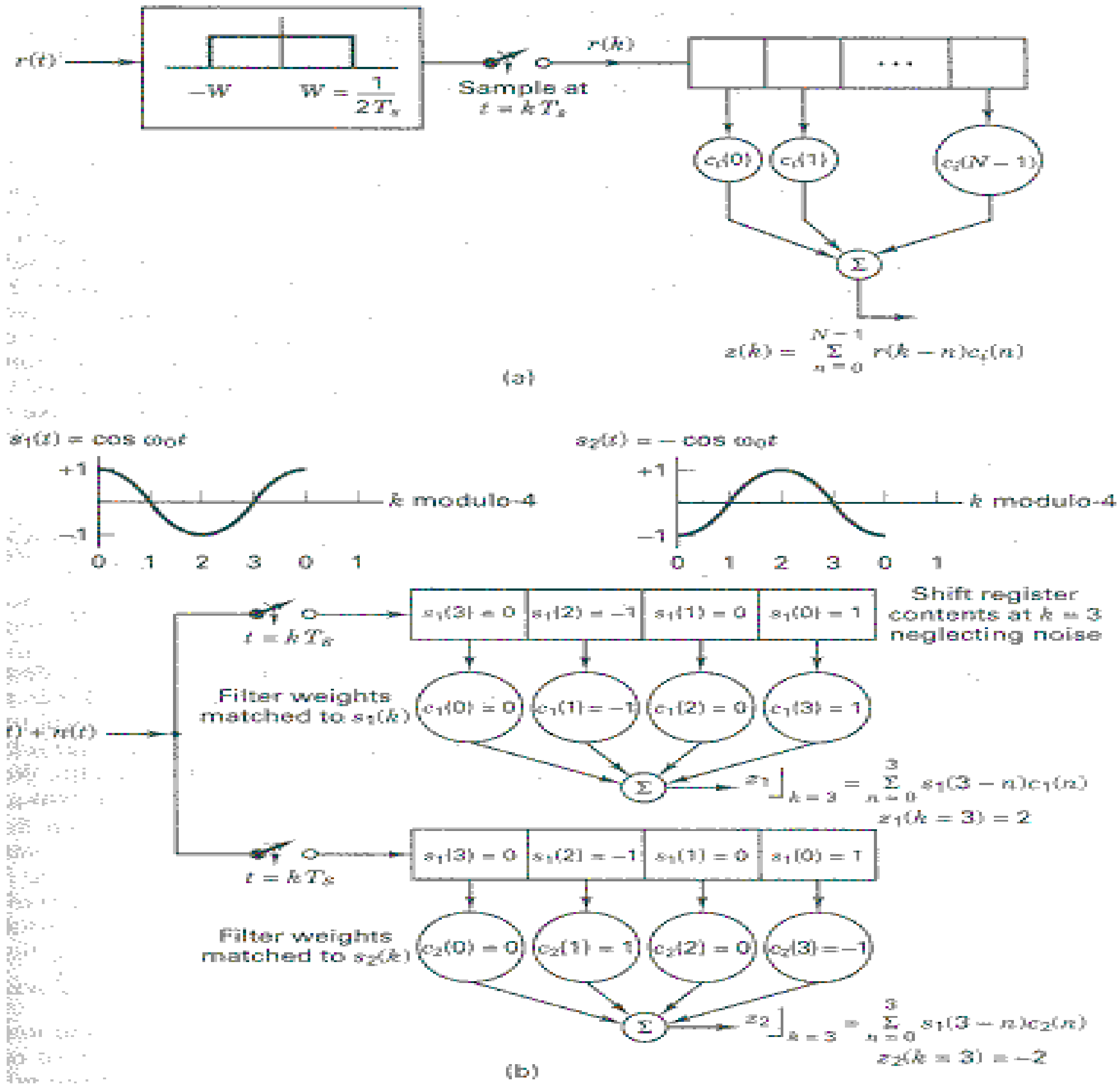


Fig 4.10

4.4.3 Coherent Detection of MPSK

- The signal space for a multiple phase-shift keying (MPSK) signal set is illustrated for a four-level (4-ary) PSK or quadriphase shift keying(QPSK)

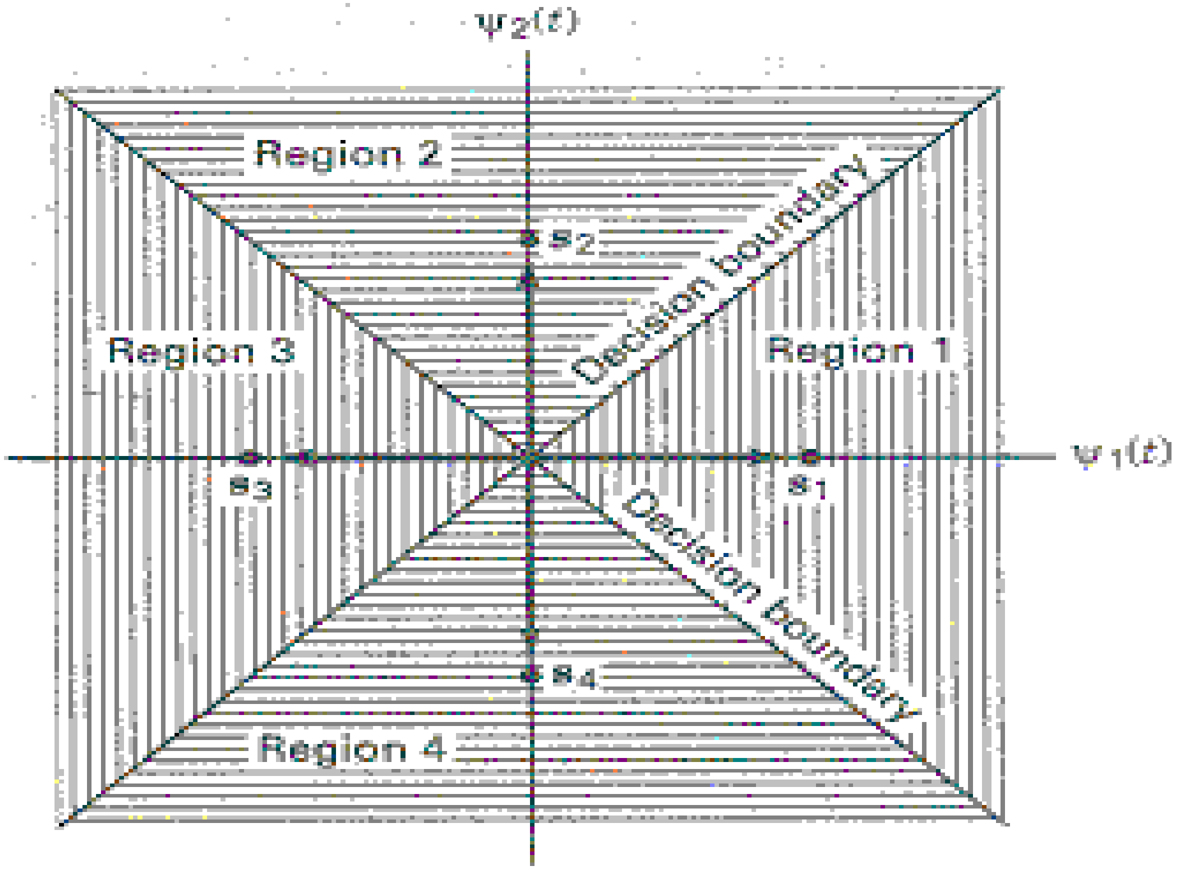


Fig 4.11

- At the transmitter, binary digits are collected two at a time for each symbol interval
- Two sequential digits instruct the modulator as to which of the four waveforms to produce
- $s_i(t)$ can be expressed as:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t - \frac{2\pi i}{M}) \qquad \begin{array}{c} 0 \le t \le T \\ i = 1,\ldots M \end{array}$$

where:

E: received energy of waveform over each symbol duration T

w_0: carrier frequency

- Assuming an ortho-normal signal space, the basis functions are:

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t \qquad \psi_2(t) = \sqrt{\frac{2}{T}} \sin \omega_0 t$$

- $s_i(t)$ can be written in terms of these orthonormal coordinates:

$$s_i(t) = a_{i1}\psi_1(t) + a_{i2}\psi_2(t) \qquad \begin{matrix} 0 \le t \le T \\ i = 1,\ldots M \end{matrix}$$

$$= \sqrt{E}\cos\left(\frac{2\pi i}{M}\right)\psi_1(t) + \sqrt{E}\sin\left(\frac{2\pi i}{M}\right)\psi_2(t)$$

- The decision rule for the detector is:
  - Decide that $s_1(t)$ was transmitted if received signal vector fall in region 1
  - Decide that $s_2(t)$ was transmitted if received signal vector fall in region 2 etc
  - i.e choose i^{th} waveform if $z_i(T)$ is the largest of the correlator outputs
- The received signal r(t) can be expressed as:

$$r(t) = \sqrt{\frac{2E}{T}}\left(\cos\phi_i \cos\omega_0 t + \sin\phi_i \sin\omega_0 t\right) + n(t) \qquad \begin{matrix} 0 \le t \le T \\ i = 1,\ldots M \end{matrix}$$

- The upper corelator computes $$X = \int_0^T r(t)\,\psi_1(t)\,dt$$
- The lower corelator computes $$Y = \int_0^T r(t)\,\psi_2(t)\,dt$$

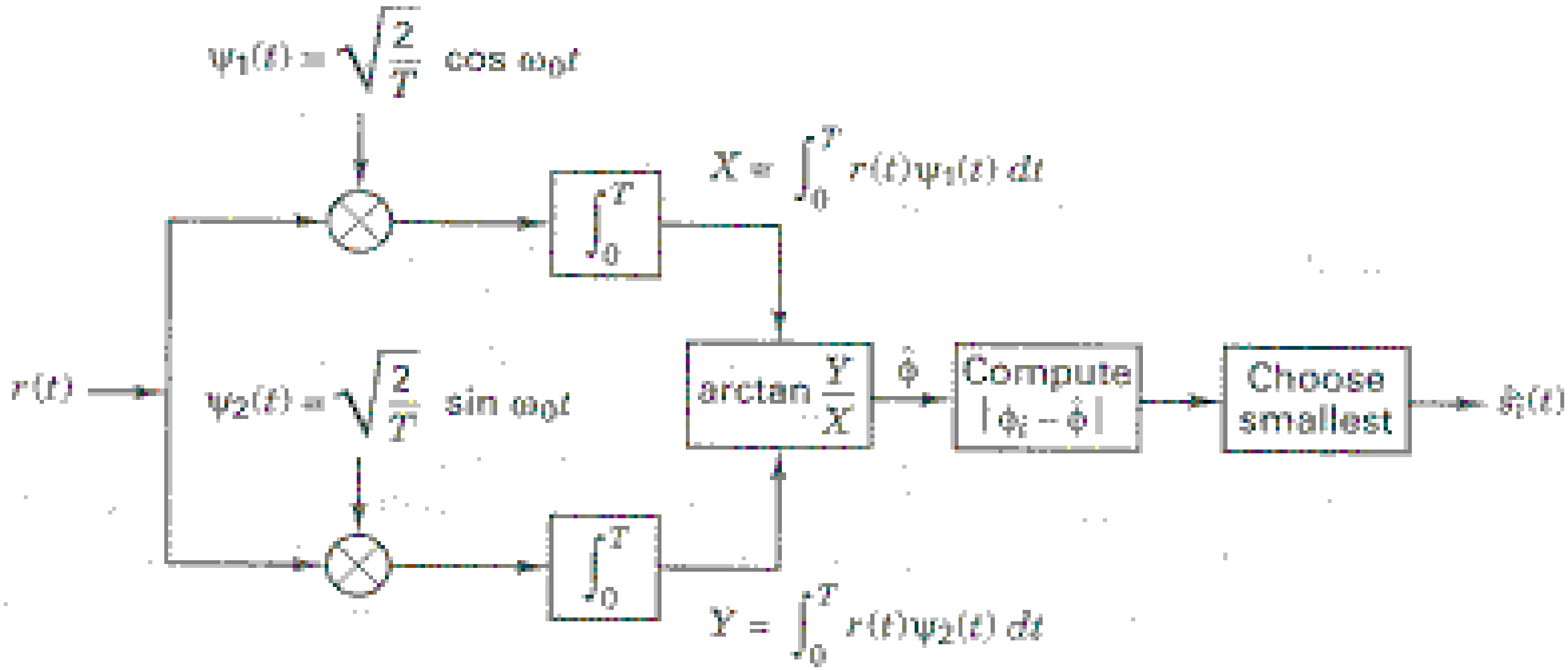


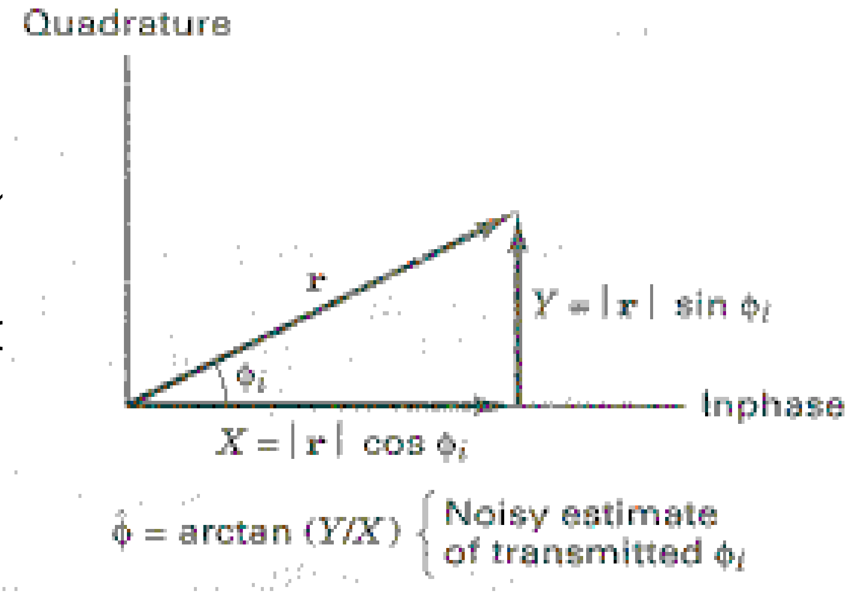
Figure 4.12 Demodulator for MPSK signals.

- The computation of the received phase angle φ can be accomplished by computing the arctan of Y/X
  Where:

  X: is the inphase component of th received signal

  Y: is the quadrature component c the received signal

  ǿ: is the noisy estimate of the transmitted φ_i



- The demodulator selects the φ_i that is closest to the angle ǿ

Fig 4.13

- Or it computes | φ_i - ǿ | for each φ_i prototypes and chooses φ_i yielding smallest output

4.4.4 Coherent Detection of FSK

- FSK modulation is characterized by the information in the frequency of the carrier
- Typical set of FSK signal waveform:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_i t + \phi) \qquad \begin{matrix} 0 \le t \le T \\ i = 1, \ldots M \end{matrix}$$

Where Φ: is an arbitrary constant

E: is the energy content of $s_i(t)$ over each symbol duration T

$(w_{i+1} - w_i)$: is typically assumed to be an integral multiple of λ/T

Assuming the basis functions form an orthonormal set:

$$\psi_j(t) = \sqrt{\frac{2}{T}} \cos \omega_j t \qquad j = 1, \ldots, N$$

- Amplitude √2/T normalizes the expected output of the MF

$$a_{ij} = \int_0^T \sqrt{\frac{2E}{T}} \cos(\omega_i t) \sqrt{\frac{2}{T}} \cos(\omega_j t)\, dt$$

- Therefore

$$a_{ij} = \begin{cases} \sqrt{E} & for\ i = j \\ 0 & otherwise \end{cases}$$

- This implies, the ith prototype signal vector is located on the ith coordinate axis at a displacement √E from the origin of the symbol space
- For general M-ary case and given E, the distance between any two prototype signal vectors s_i and s_j is constant:

$$d(s_i, s_j) = \| s_i - s_j \| = \sqrt{2E} \qquad for\ i \neq j$$

Signal space partitioning for 3-ary FSK

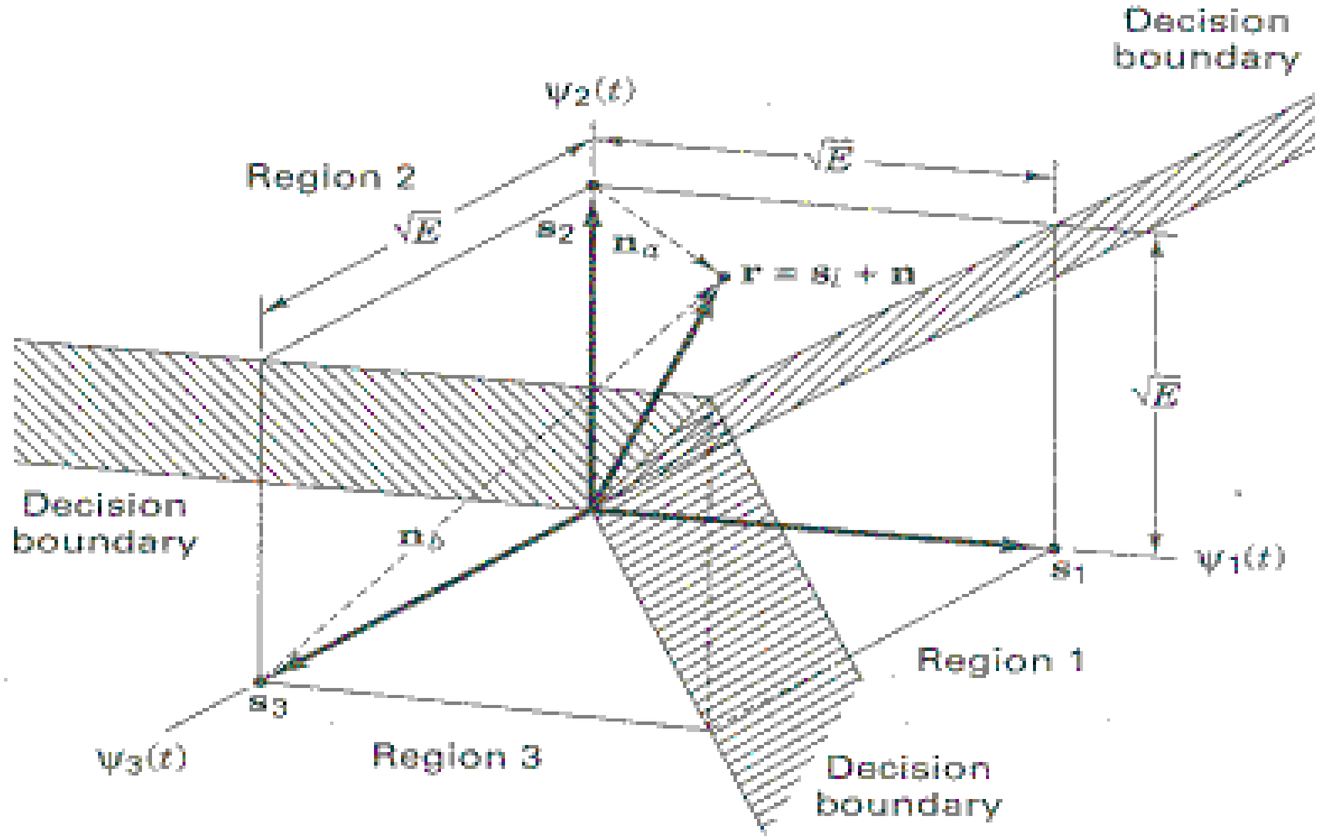


Figure 4.14 Partitioning the signal space for a 3-ary FSK signal.