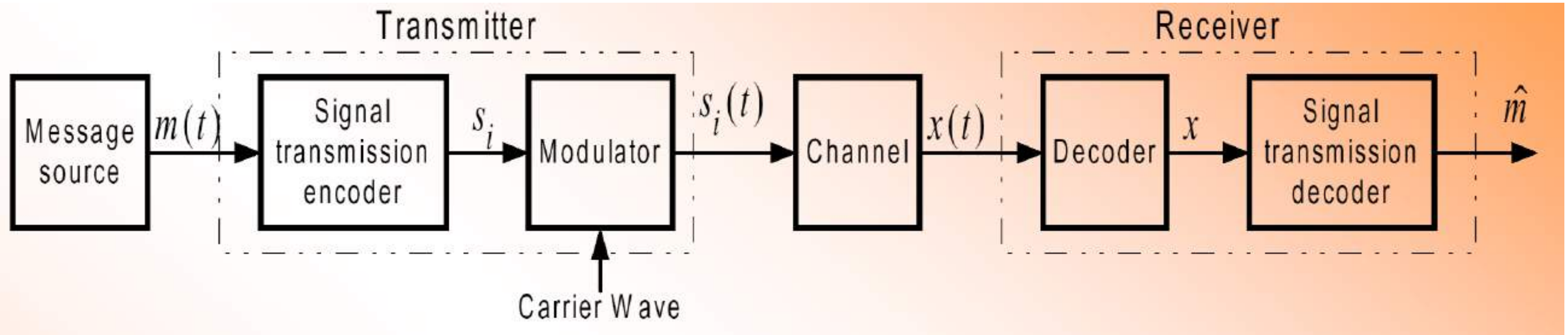
Coherent & Non Coherent Detection

Bandpass Modulation and Demodulation



- Bandpass Modulation is the process by which some characteristics of a sinusoidal waveform is varied according to the message signal.
- ***Modulation*** shifts the spectrum of a baseband signal to some high frequency.
- Demodulator/Decoder baseband waveform recovery

Why Modulate?

- Most channels require that the baseband signal be shifted to a higher frequency
- For example in case of a wireless channel antenna size is inversely proportional to the center frequency, this is difficult to realize for baseband signals.
  - For speech signal f = 3 kHz $\Rightarrow \lambda=c/f=(3x10^{8)}/(3x10^{3})$
  - Antenna size without modulation $\lambda/4=10^{5}/4$ meters = 15 miles - practically unrealizable
  - Same speech signal if amplitude modulated using f_c=900MHz will require an antenna size of about 8cm.
  - This is evident that efficient antenna of realistic physical size is needed for radio communication system
- Modulation also required if channel has to be shared by several transmitters (Frequency division multiplexing).

4.2 Digital Bandpass Modulation Techniques

Three ways of representing bandpass signal:

- **(1) Magnitude and Phase (M & P)**
  - Any bandpass signal can be represented as:

$$s(t) = A(t)\cos[\theta(t)] = A(t)\cos[\omega_0 t + \phi(t)]$$

    - *A(t)* ≥ *0* is real valued signal representing the magnitude
    - Θ(t) is the genarlized angle
    - φ*(t)* is the phase

  - The representation is easy to interpret physically, but often is not mathematically convenient
  - In this form, the modulated signal can represent information through changing three parameters of the signal namely:
    - Amplitude *A(t)* : *as in* Amplitude Shift Keying (**ASK**)
    - Phase φ*(t)* : *as in* Phase Shift Keying (**PSK**)
    - Frequency *d*Θ*(t)/ dt* : *as in* Frequency Shift Keying (**FSK**)

Angle Modulation

- Consider a signal with constant frequency:

$$s(t) = A(t)\cos(\theta(t)) = A(t)\cos(\omega_0 t + \varphi)$$

- Its instantaneous frequency can be written as:

$$\omega_i(t) = \frac{d\theta(t)}{dt} = \omega_0$$

or

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)d\alpha$$

Phase Shift Keying (PSK) or PM

- Consider a message signal m(t), we can write the phase modulated signal as

$$\theta(t) = \omega_c t + K_p m(t)$$

$$s_{PM}(t) = A\cos[\omega_c t + K_p m(t)]$$

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + K_p m'(t)$$

Frequency Shift Keying (FSK) or FM

- In case of Frequency Modulation

$$\omega_i(t) = \omega_0 + K_f m(t)$$

$$\theta(t) = \int_{-\infty}^{t} [\omega_0 + K_f m(t)] d\alpha$$

$$= \omega_0 t + K_f \int_{-\infty}^{t} m(\alpha) d\alpha$$

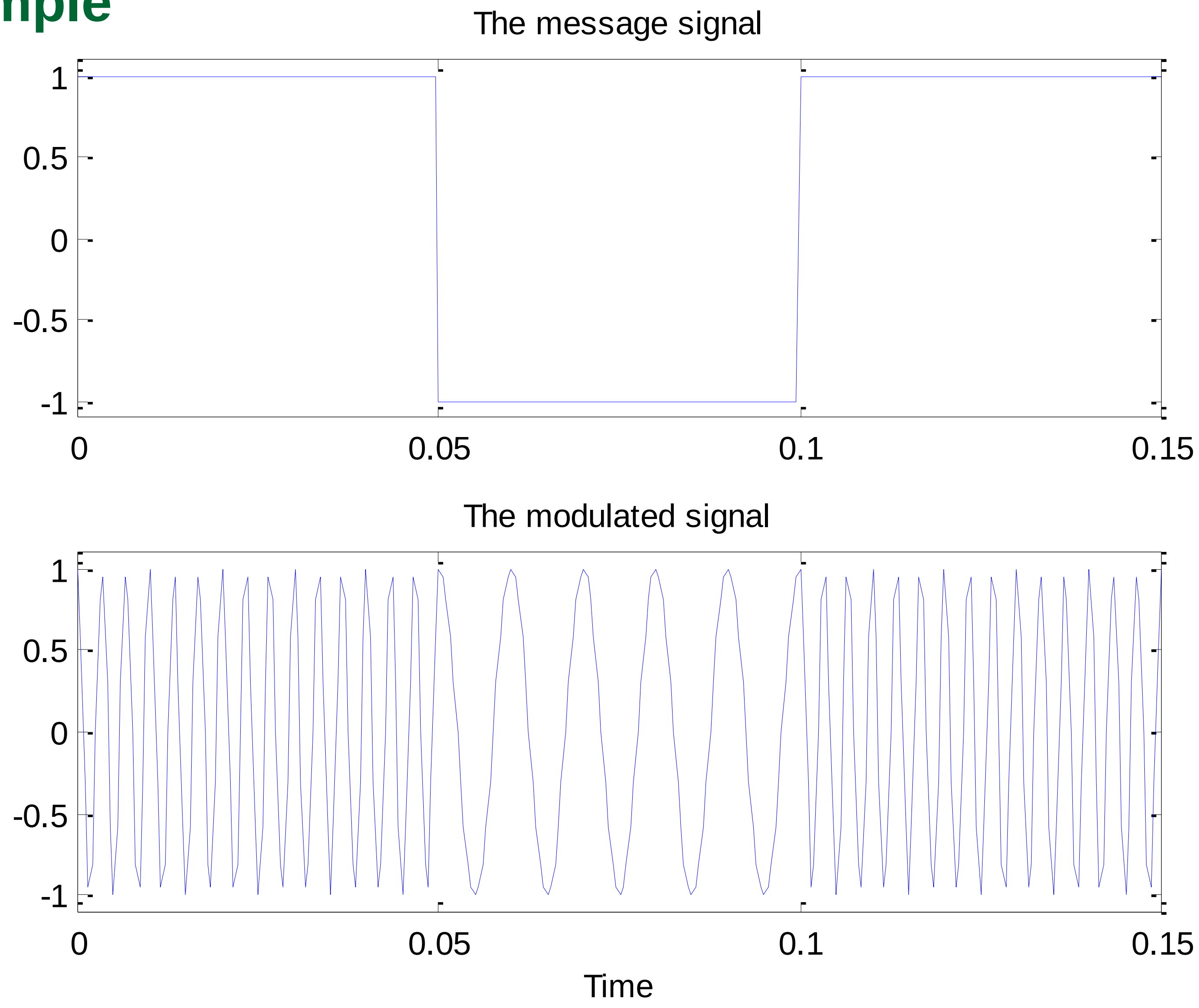
$$s_{FM}(t) = A\cos[\omega_0 t + K_f \int_{-\infty}^{t} m(\alpha) d\alpha]$$

$$= A\cos[\omega_0 t + K_f a(t)]$$

where:

$$a(t) = \int_{-\infty}^{t} m(\alpha) d\alpha$$

Example

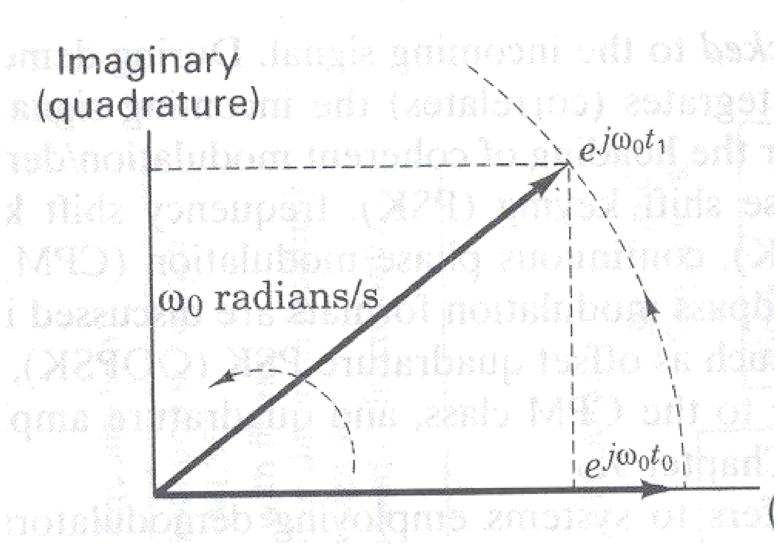

The message signal
1
0.5
0
-0.5
-1
0
0.05
0.1
0.15
The modulated signal
1
0.5
0
-0.5
-1
0
0.05
0.1
0.15
Time

4.2.1 Phasor Representation of Sinusoid

- Consider the trigonometric identity called the Euler's theorem:

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

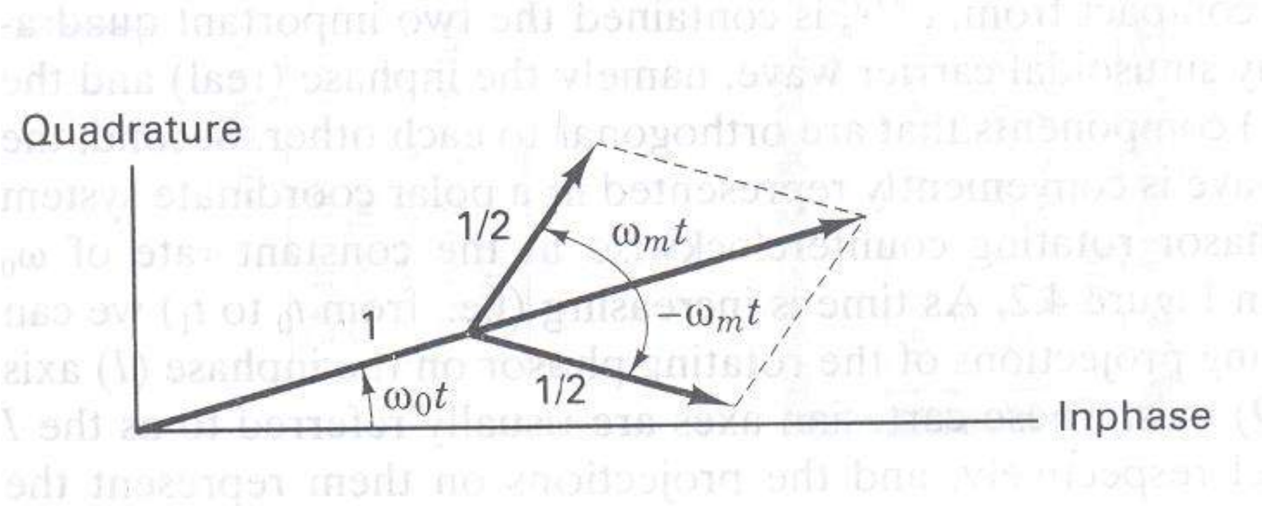
- Using this identity we can have the phasor representation of the sinusoids. Figure 4.2 below shows such relation:



Phasor Representation of Amplitude Modulation

- Consider the AM signal in phasor form:

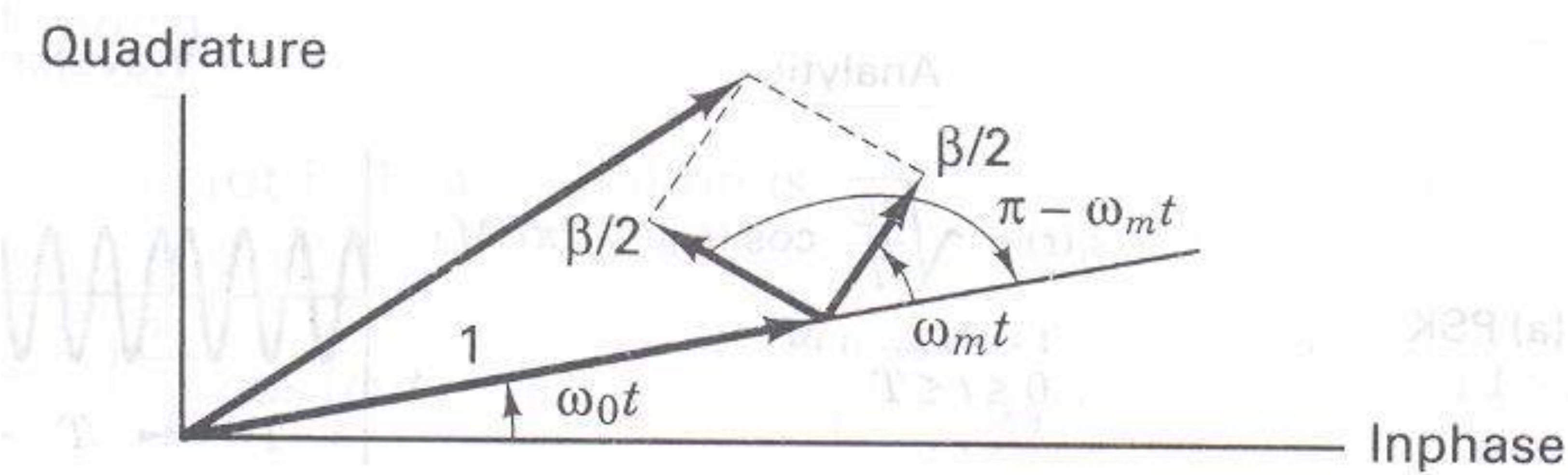
$$s(t) = \mathrm{Re}\left\{ e^{j\omega_0 t}\left(1 + \frac{e^{j\omega_m t}}{2} + \frac{e^{-j\omega_m t}}{2}\right)\right\}$$



Phasor Representation of FM

- Consider the FM signal in phasor form:

$$s(t) = \mathrm{Re}\left\{ e^{j\omega_0 t} \left(1 - \frac{\beta}{2} e^{-j\omega_m t} + \frac{\beta}{2} e^{j\omega_m t} \; \frac{e^{-j\omega_m t}}{2} \right) \right\}$$



Digital Modulation Schemes

- Basic Digital Modulation Schemes:
  - Amplitude Shift Keying (ASK)
  - Frequency Shift Keying (FSK)
  - Phase Shift Keying (PSK)
  - Amplitude Phase Keying (APK)

- For Binary signals $(M = 2)$, we have
  - Binary Amplitude Shift Keying (BASK)
  - Binary Phase Shift Keying (BPSK)
  - Binary Frequency Shift Keying (BFSK)

- For $M > 2$, many variations of the above techniques exit usually classified as M-ary Modulation/detection

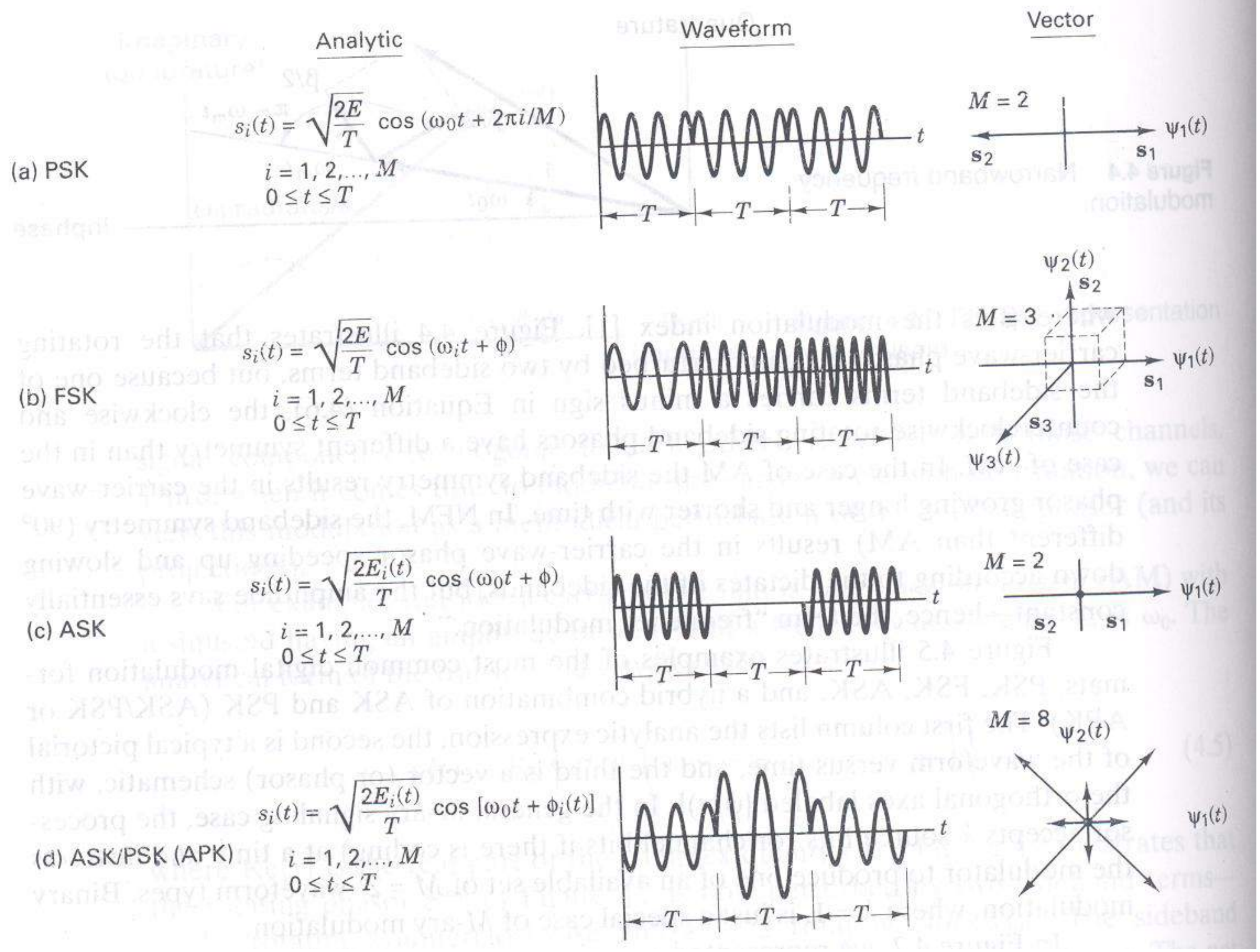
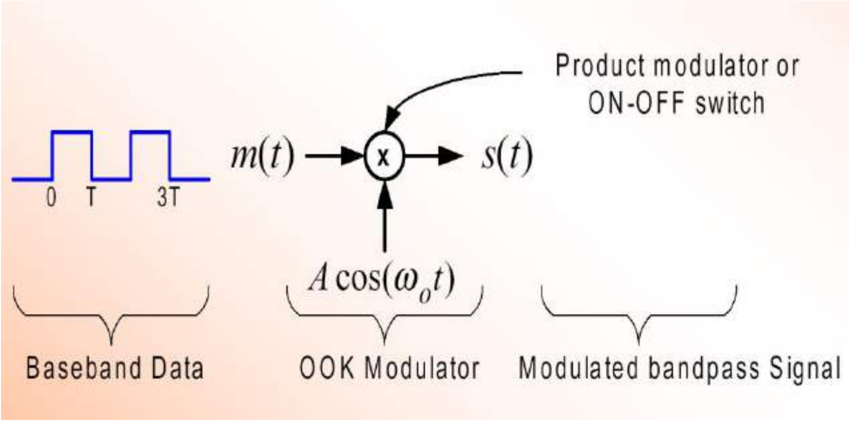
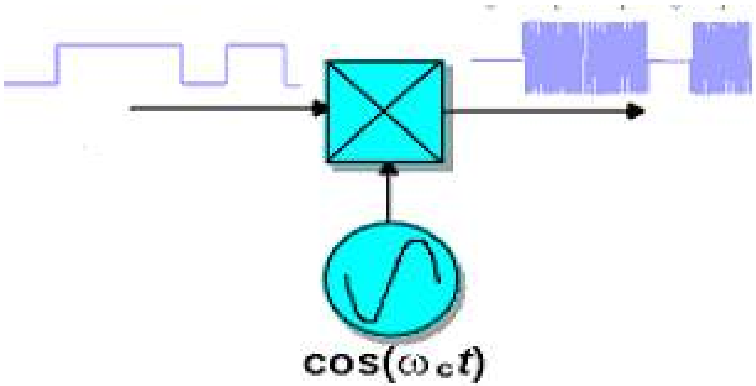


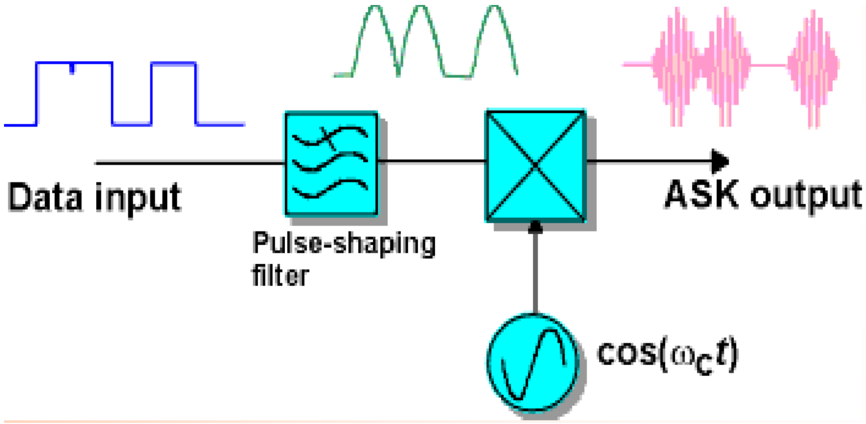
Figure4.5: digital modulations, (a) PSK (b) FSK (c) ASK (d) ASK/PSK (APK)

Amplitude Shift Keying

- **Modulation Process**
  - In Amplitude Shift Keying (***ASK)***, the amplitude of the carrier is switched between two (or more) levels according to the digital data
  - For BASK (also called ***ON-OFF Keying (OOK)***), one and zero are represented by two amplitude levels A_1 and A_0







- ***Analytical Expression:***

$$s(t) = \begin{cases} A_i \cos(\omega_c t), & 0 \le t \le T \; binary\, 1 \\ 0, & 0 \le t \le T \; binary\; 0 \end{cases}$$

where A_i = peak amplitude

$$s(t) = A\cos(\omega_0 t) = \sqrt{2}A_{rms}\cos(\omega_0 t) = \sqrt{2A_{rms}^2}\cos(\omega_0 t)$$

$$= \sqrt{2P}\cos(\omega_0 t) = \sqrt{\frac{2E}{T}}\cos(\omega_0 t) \;\rightarrow P = \frac{V^2}{R}$$

Hence,

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_i(t)}{T}}\cos(\omega_i t), & 0 \le t \le T \; binary\, 1 \\ 0, & 0 \le t \le T \; binary\; 0 \end{cases}, \; i = 0,2,......M-1$$

where

$$E = \int_0^T s_i^2(t)dt, \; i = 0,2,......M-1$$

- Where for binary ASK (also known as ON OFF Keying (OOK))

$$s_1(t) = A_c m(t)\cos(\omega_c t + \phi), \quad 0 \le t \le T \; binary\,1$$
$$s_0(t) = 0, \qquad\qquad\qquad\qquad 0 \le t \le T \; binary\,0$$

- **Mathematical ASK Signal Representation**
  - The ***complex envelope*** of an ASK signal is:

$$g(t) = A_c m(t)$$

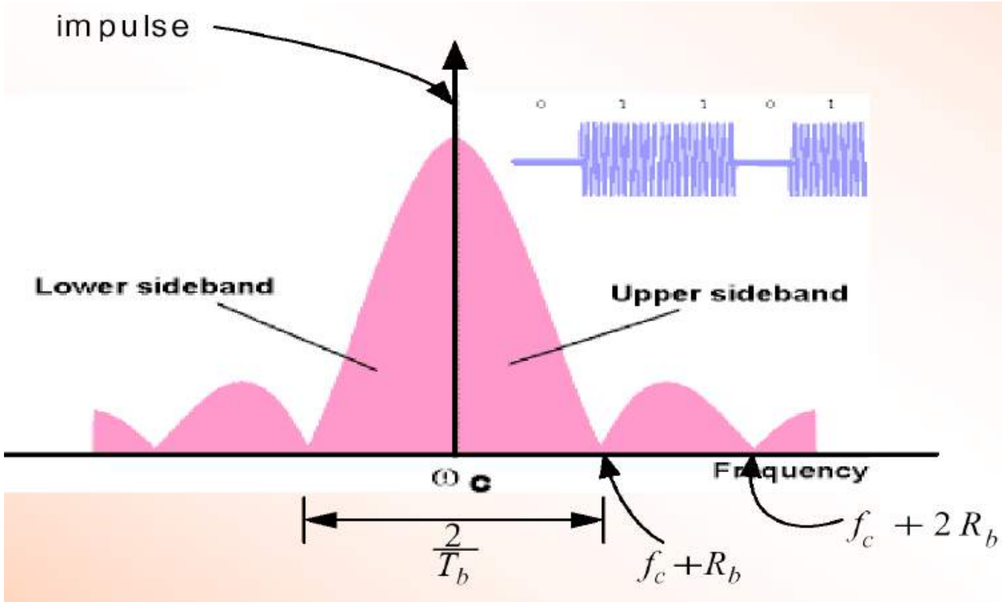
  - The ***magnitude*** and ***phase*** of an ASK signal are:

$$A(t) = A_c m(t), \qquad \phi(t) = 0$$

  - The **in-phase** and **quadrature** components are:

$$x(t) = A_c m(t)$$

$y(t) = 0,$ the quadrature component is wasted.



It can be seen that the bandwidth of ASK modulated is twice that occupied by the source baseband stream

- **Bandwidth of ASK**
  - Bandwidth of ASK can be found from its power spectral density
  - The bandwidth of an ASK signal is twice that of the unipolar NRZ line code used to create it., i.e.,

$$B = 2R_b = \frac{2}{T_b}$$

- This is the **null-to-null bandwidth** of ASK

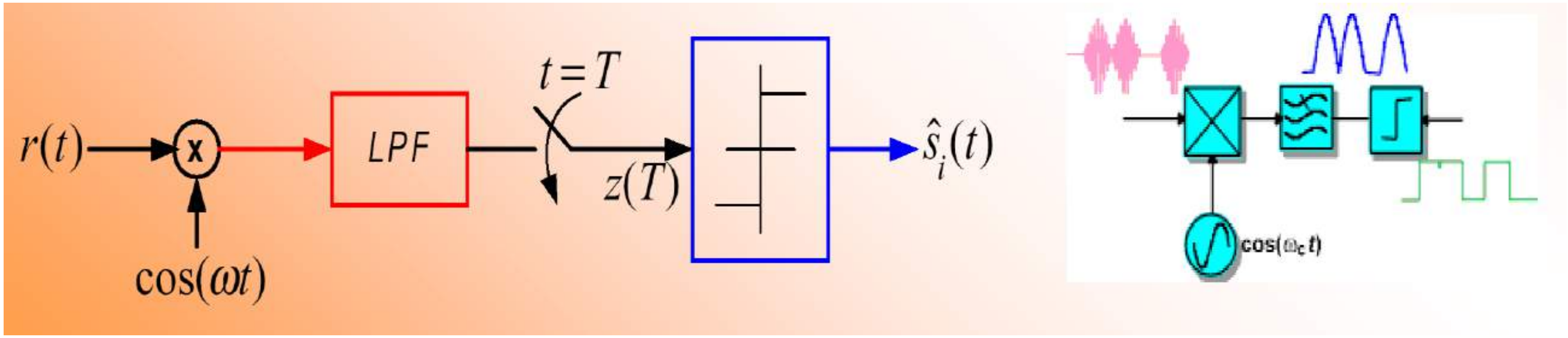
- If raised cosine rolloff pulse shaping is used, then the bandwidth is:

$$B = (1+r)R_b \Rightarrow W = \frac{1}{2}(1+r)R_b$$

- Spectral efficiency of ASK is half that of a baseband unipolar NRZ line code
  - This is because the quadrature component is wasted
- 95% energy bandwidth

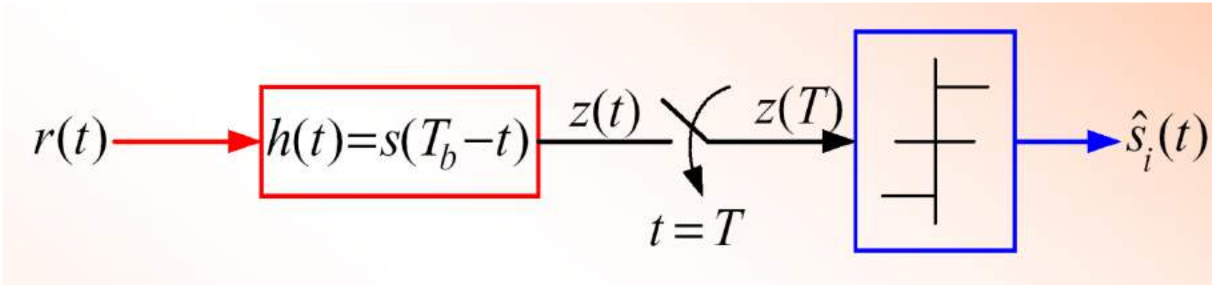
$$B = \frac{3}{T_b} = 3R_b$$

Detectors for ASK
Coherent Receiver

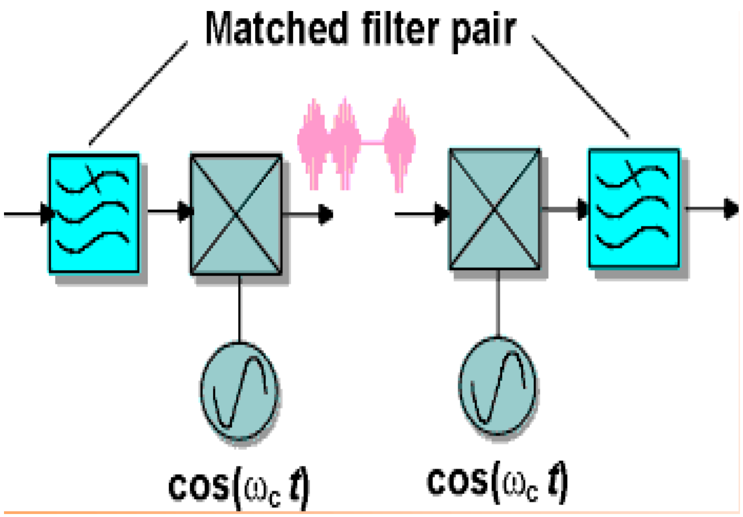


- Coherent detection requires the phase information
- A coherent detector mixes the incoming signal with a locally generated carrier reference
- Multiplying the received signal *r(t)* by the receiver local oscillator (say $A_c cos(w_c t)$) yields a signal with a baseband component plus a component at $2f_c$
- Passing this signal through a low pass filter eliminates the high frequency component
  - In practice an integrator is used as the LPF

- The output of the LPF is sampled once per bit period
- This sample *z(T)* is applied to a decision rule
  - *z(T)* is called the **decision statistic**
- Matched filter receiver of OOK signal



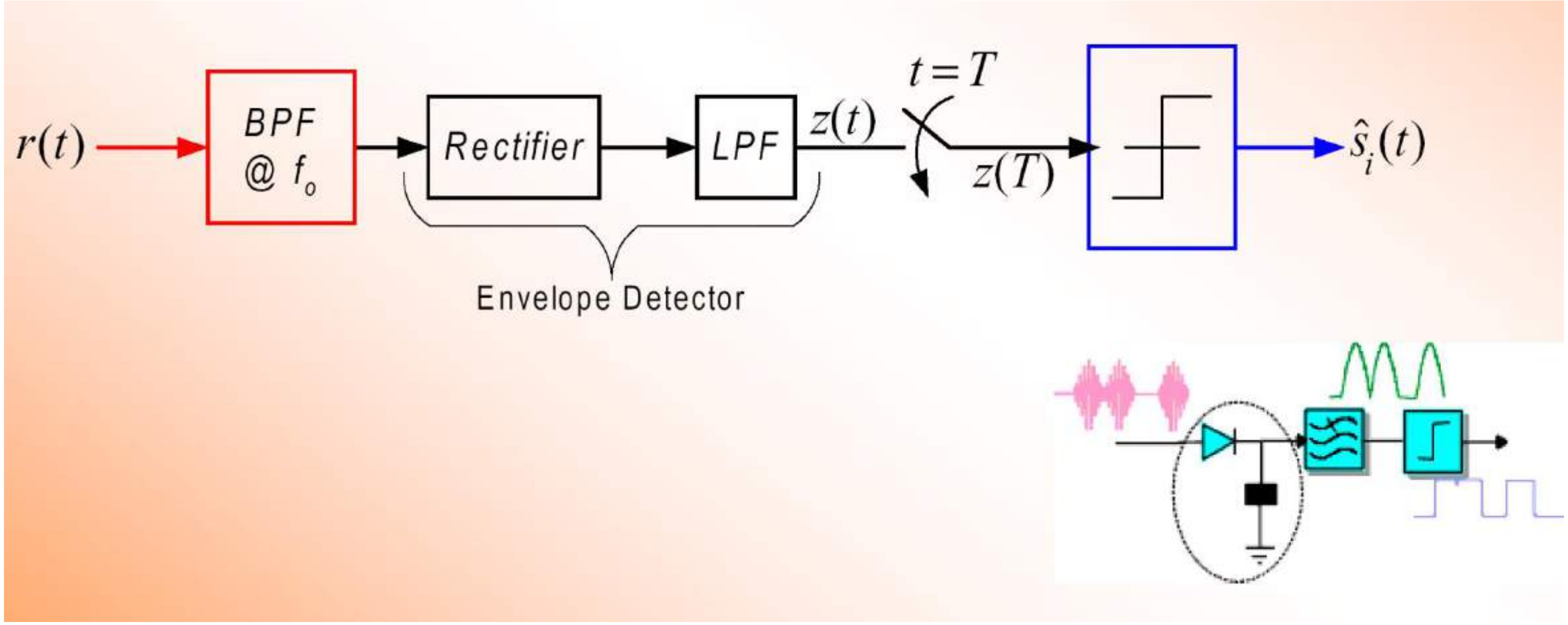
- A MF pair such as the root raised cosine filter can thus be used to shape the source and received baseband symbols
- In fact this is a very common approach in signal detection in most bandpass data modems



Noncoherent Receiver

- Does not require a phase reference at the receiver
- If we do not know the phase and frequency of the carrier, we can use a ***noncoherent receiver* to recover AS**K signal

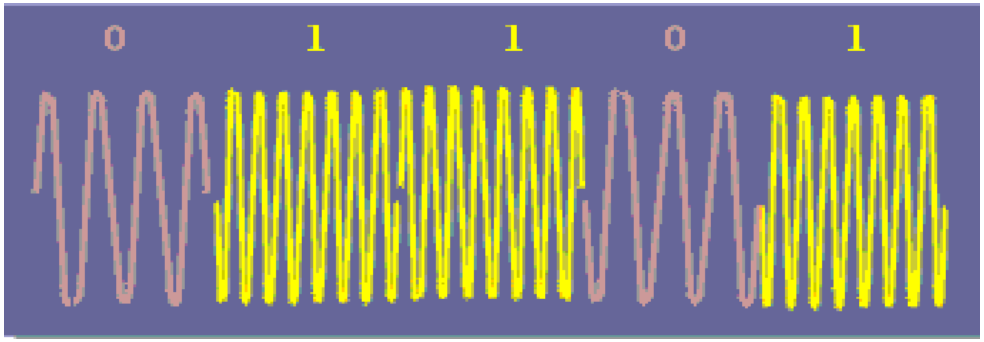
- ***Envelope Detector*:**



- The simplest implementation of an envelope detector comprises a diode rectifier and smoothing filter

Frequency Shift Keying (FSK)

- In *FSK*, the instantaneous carrier frequency is switched between 2 or more levels according to the baseband digital data
  - data bits select a carrier at one of two frequencies
  - the data is encoded in the frequency
- Until recently, FSK has been the most widely used form of digital modulation;Why?
  - Simple both to generate and detect
  - Insensitive to amplitude fluctuations in the channel
- FSK conveys the data using distinct carrier frequencies to represent symbol states
- An important property of FSK is that the *amplitude of the modulated wave is constant*
- **Waveform**



- **Analytical Expression**

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos(\underbrace{\omega_i t + \phi}), \quad i = 0,1,\ldots.M-1$$

$$\left.\begin{aligned} \theta_i(t) &= [\omega_0 t + \omega_d \int_{-\infty}^{t} m(\tau)d\tau] \\ f_i &= \frac{d}{dt}\theta_i(t) = f_0 + f_d m(t) \end{aligned}\right\} \text{Analog form}$$

- General expression is

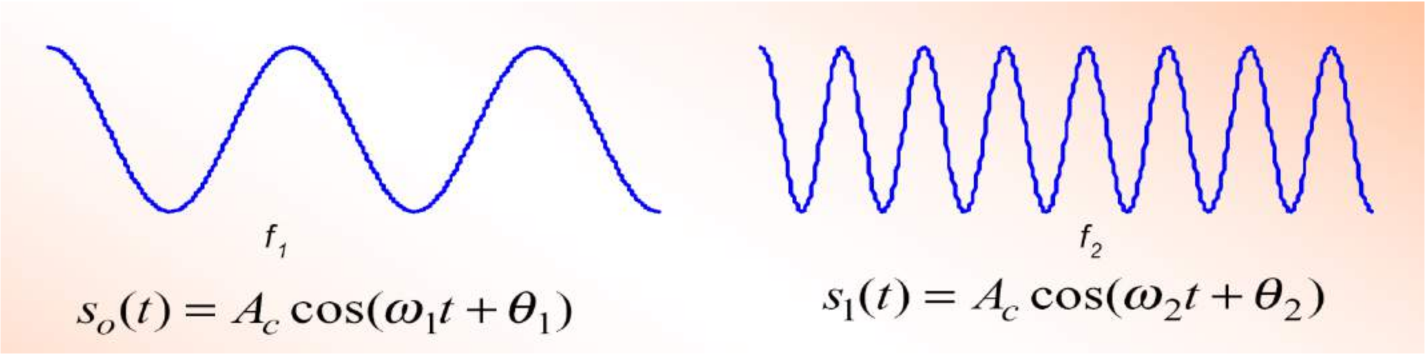
$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_0 t + 2\pi i \Delta f t), \quad i = 0,1,\ldots.M-1$$

Where $\Delta f = f_i - f_{i-1}$

$$f_i = f_0 + i\Delta f \quad and \quad E_s = kE_b, \; T_s = kT_b$$

Binary FSK

- In ***BFSK***, 2 different frequencies, *f1* and *f2* = *f1* + Δ *f* are used to transmit binary information



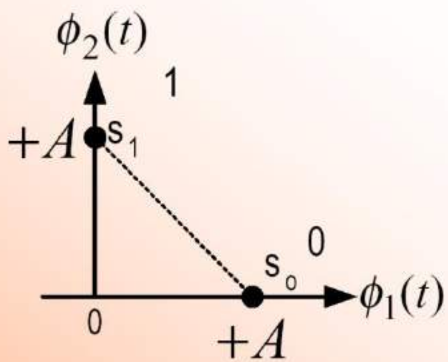
- Data is encoded in the frequencies
- That is, *m(t)* is used to select between 2 frequencies:
- *f1* is the mark frequency, and *f2* is the space frequency

$$s_0(t) = \sqrt{\frac{2E_s}{T_b}} \cos 2\pi(f_1 + \theta_1), \quad 0 \le t \le T_b$$

$$s_1(t) = \sqrt{\frac{2E_s}{T_b}} \cos 2\pi(f_2 + \theta_2), \quad 0 \le t \le T_b$$

$$s(t) = \begin{cases} A_c \cos(\omega_1 t + \theta_1), & when\ m(t) = +1\ or\ X_n = 1 \\ A_c \cos(\omega_2 t + \theta_2), & when\ m(t) = -1\ or\ X_n = 0 \end{cases}$$

- **Binary Orthogonal Phase FSK**



$$\phi_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_1 t + \theta_1)$$

$$\phi_2(t) = \sqrt{\frac{2E}{T}} \cos(\omega_2 t + \theta_2)$$

- When w_0 an w_1 are chosen so that $f_1(t)$ and $f_2(t)$ are orthogonal, i.e.,

$$\int_{-\infty}^{\infty} \phi_1(t)\phi_2(t) = 0$$

  - form a set of *K = 2* basis orthonormal basis functions

Phase Shift Keying (PSK)

- General expression is

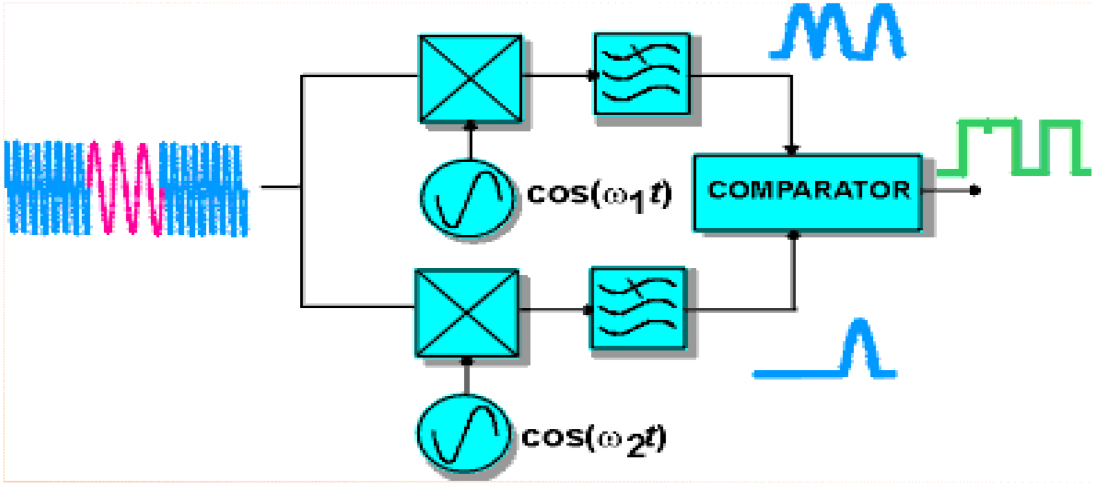
$$s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos[2\pi f_0 t + \phi_i(t)], \quad i = 0,1,\ldots.M-1$$

- Where

$$\phi_i(t) = \frac{2\pi i}{M} \quad i = 0,1,\ldots.M-1$$

3. Coherent Detection of Binary FSK

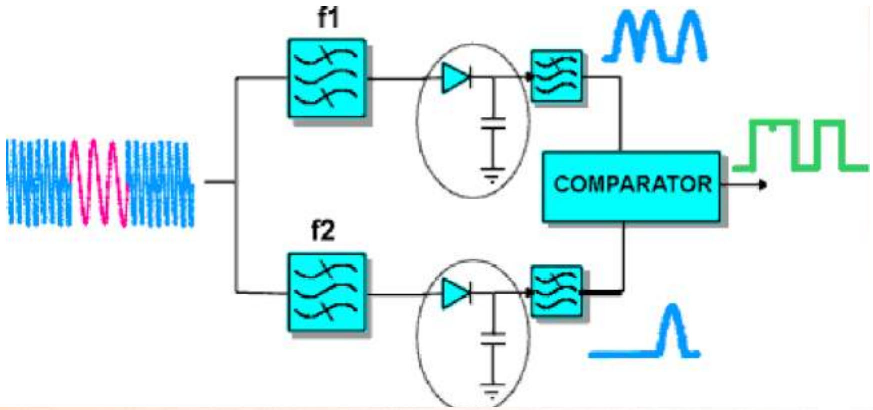
- Coherent detection of Binary FSK is similar to that for ASK but in this case there are 2 detectors tuned to the 2 carrier frequencies

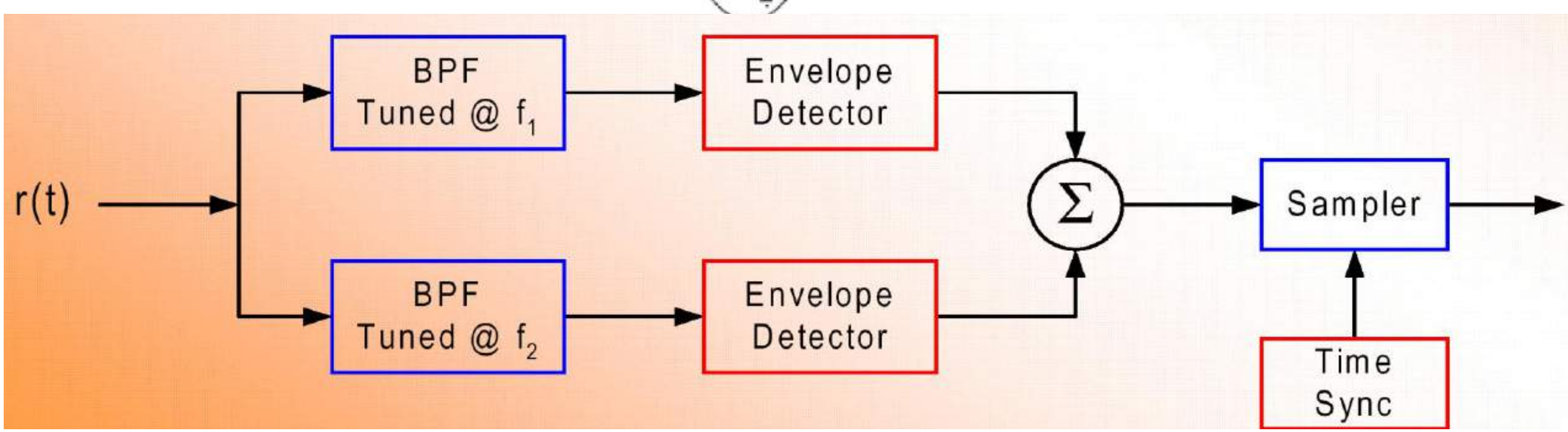


- Recovery of *fc* in receiver is made simple if the frequency spacing between symbols is made equal to the ***symbol rate.***

Non-coherent Detection

- One of the simplest ways of detecting binary FSK is to pass the signal through 2 BPF tuned to the 2 signaling freqs and detect which has the larger output averaged over a symbol period

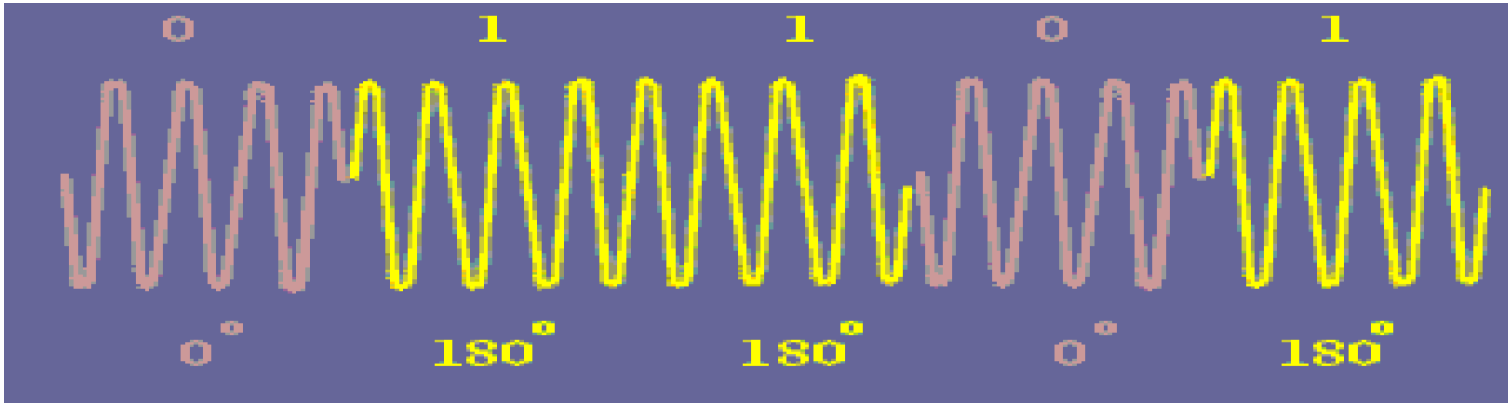




Phase Shift Keying (PSK)

- In PSK, the phase of the carrier signal is switched between 2 (for BPSK) or more (for MPSK) in response to the baseband digital data
- With PSK the information is contained in the instantaneous phase of the modulated carrier
- Usually this phase is imposed and measured with respect to a fixed carrier of known phase – Coherent PSK
- For binary PSK, phase states of 0^o and 180^o are used

- **Waveform:**



- Analytical expression can be written as

$$s_i(t) = A\, g(t) \cos[\omega_c t + \phi_i(t)], \qquad 0 \le t \le T_b,\ \ i = 1, 2, \ldots., M$$

where

- g(t) is signal pulse shape
- A = amplitude of the signal
- ø = carrier phase

- The range of the carrier phase can be determined using

$$\phi_i(t) = \frac{2\pi(i-1)}{M} \qquad i = 1, \ldots. M$$

- For a rectangular pulse, we obtain

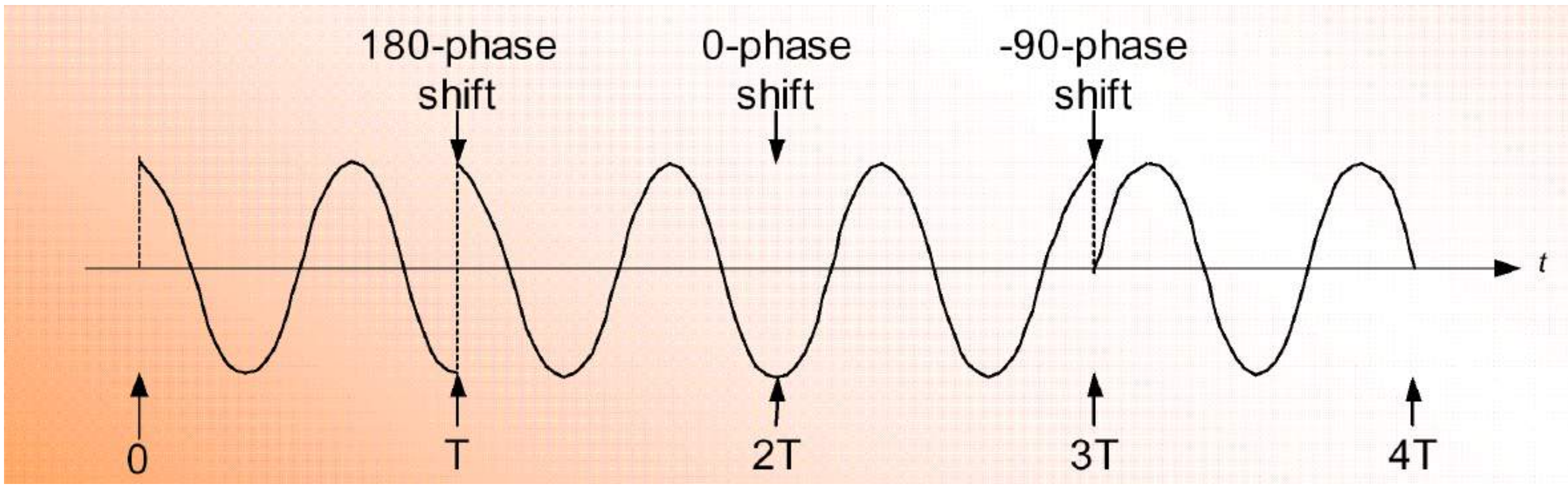
$$g(t) = \sqrt{\frac{2}{T_b}}, \quad 0 \le t \le T_b;\ \ and\ assume\ A = \sqrt{E_b}$$

- We can now write the analytical expression as

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(\omega_c t + \frac{2\pi(i-1)}{M}\right), \quad 0 \le t \le T_b, \ and \ i = 1,2,....M$$

Constant envelope

carrier phase changes abruptly at the beginning of each signal interval



- In PSK the carrier phase changes abruptly at the beginning of each signal interval while the amplitude remains constant