

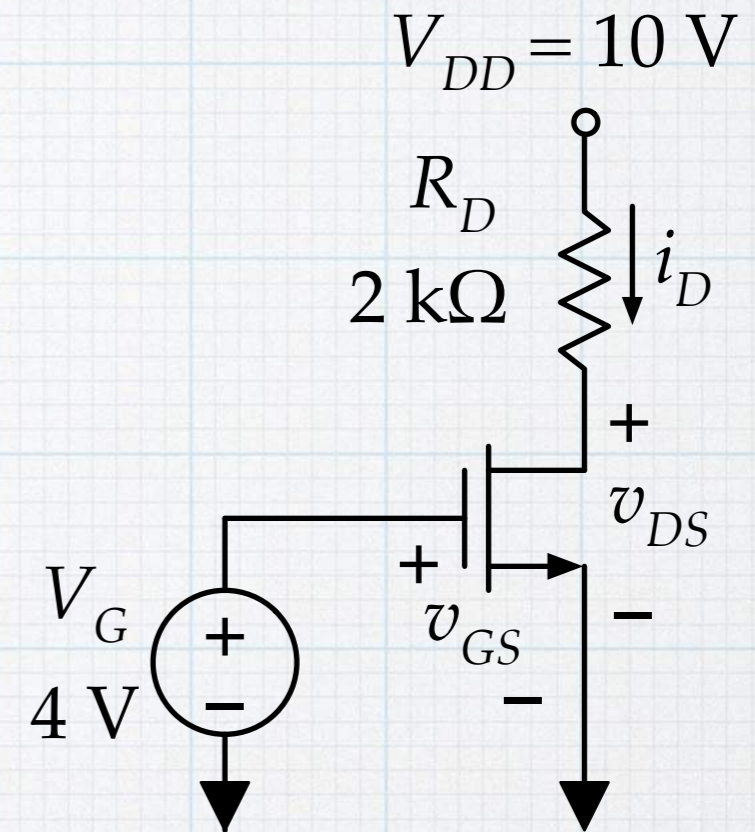
NMOS examples

For the circuit shown, use the the NMOS equations to find i_D and v_{DS} .

For the NMOS, $V_T = 1.5 \text{ V}$ and $K = 0.5 \text{ mA/V}^2$.

$$v_{GS} = V_G = 4 \text{ V} \rightarrow \text{the NMOS is on.}$$

Assume that the transistor is in saturation.



$$i_D = K (v_{GS} - V_T)^2 = (0.5 \text{ mA/V}^2) [4 \text{ V} - 1.5 \text{ V}]^2 = 3.125 \text{ mA}$$

$$v_{DS} = V_{DD} - i_D R_D = 10 \text{ V} - (3.125 \text{ mA}) (2 \text{ k}\Omega) = 3.75 \text{ V}$$

$$v_{GS} - V_T = 4 \text{ V} - 1.5 \text{ V} = 2.5 \text{ V}$$

$$v_{DS} > v_{GS} - V_T \rightarrow \text{saturation confirmed. Q.E.D.}$$

Example 2

For the circuit shown, use the the NMOS equations to find i_D and v_{DS} .

For the NMOS, $V_T = 1.0 \text{ V}$ and $K = 0.5 \text{ mA/V}^2$.

Since $V_G > V_T \rightarrow$ the NMOS is on.

Guess that the transistor is in saturation.

$$i_D = K (v_{GS} - V_T)^2$$

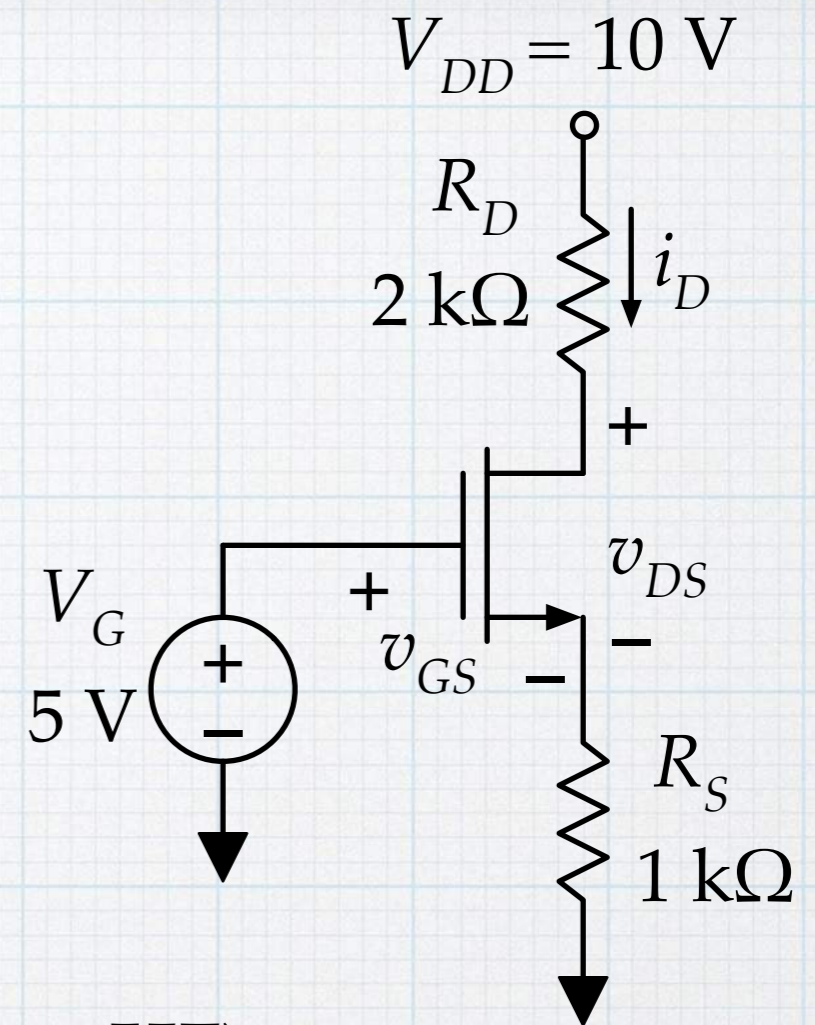
$$v_{GS} = V_G - i_S R_S \text{ (and } i_S = i_D, \text{ as always for a FET)}$$

$$i_D = K (V_G - i_D R_S - V_T)^2$$

$$= K \left[R_S^2 i_D^2 - 2 (V_G - V_T) R_S i_D + (V_G - V_T)^2 \right]$$

Re-arranging:

$$i_D^2 - \left[\frac{1}{KR_S^2} + 2 \left(\frac{V_G - V_T}{R_S} \right) \right] i_D + \left[\frac{V_G - V_T}{R_S} \right]^2 = 0$$



$$i_D^2 - \left[\frac{1}{KR_S^2} + 2 \left(\frac{V_G - V_T}{R_S} \right) \right] i_D + \left[\frac{V_G - V_T}{R_S} \right]^2 = 0$$

Plug in the numbers:

$$i_D^2 - [10 \text{ mA}] i_D + [9 \text{ mA}^2] = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic equation:

$$i_D = 9 \text{ mA} \text{ or } i_D = 1 \text{ mA.}$$

or, if $a = 1$

$$x = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

Which is right? Check the v_{GS} for both.

If $i_D = 9 \text{ mA}$, $v_{GS} = V_G - i_D R_S = 4 \text{ V} - 9 \text{ V} = -5 \text{ V}$. No way! The NMOS would not be on in that case. This root cannot be correct.

If $i_D = 1 \text{ mA}$, $v_{GS} = V_G - i_D R_S = 4 \text{ V} - 1 \text{ V} = 3 \text{ V}$. OK, this is consistent.

Finally, $v_{GS} - V_T = V_G - i_D R_S - V_T = 2 \text{ V}$, and $v_{DS} = V_{DD} - i_D R_D - i_D R_S = 7 \text{ V}$.

$v_{DS} > v_{GS} - V_T \rightarrow$ saturation confirmed.

Example 3

For the circuit shown, use the the NMOS equations to find i_D and v_{DS} .

For the NMOS, $V_T = 1.5 \text{ V}$ and $K = 0.25 \text{ mA/V}^2$.

First note that $v_{GS} = v_{DS}$, so the NMOS *must* be in saturation. ($v_{DS} > v_{GS} - V_T$) No guess needed.

Since $V_{DD} > V_T \rightarrow$ the NMOS is on.

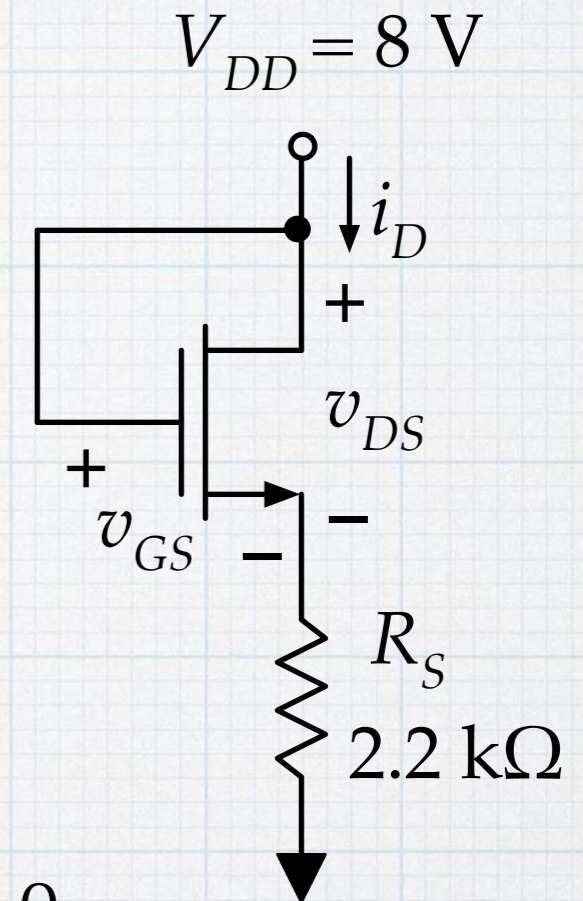
And $i_S = i_D$ (always for a FET) and don't forget that $i_G = 0$.

$$i_D = K (v_{GS} - V_T)^2 \quad v_{GS} = V_{DD} - i_S R_S$$

$$\begin{aligned} i_D &= K (V_{DD} - i_D R_S - V_T)^2 \\ &= K \left[R_S^2 i_D^2 - 2 (V_{DD} - V_T) R_S i_D + (V_{DD} - V_T)^2 \right] \end{aligned}$$

(Same basic form as Example 2.)

$$i_D^2 - \left[\frac{1}{K R_S^2} + 2 \left(\frac{V_{DD} - V_T}{R_S} \right) \right] i_D + \left[\frac{V_{DD} - V_T}{R_S} \right]^2 = 0$$



$$i_D^2 - \left[\frac{1}{KR_S^2} + 2 \left(\frac{V_{DD} - V_T}{R_S} \right) \right] i_D + \left[\frac{V_{DD} - V_T}{R_S} \right]^2 = 0$$

Plug in the numbers:

$$i_D^2 - [6.74 \text{ mA}] i_D + [8.73 \text{ mA}^2] = 0$$

Use the quadratic equation:

$$i_D = 4.99 \text{ mA} \text{ or } i_D = 1.75 \text{ mA}.$$

Which is right? Check the v_{GS} for both.

If $i_D = 4.99 \text{ mA}$, $v_{GS} = V_{DD} - i_D R_S = 8 \text{ V} - 10.98 \text{ V} = -2.98 \text{ V}$. Nope – the NMOS would not be on in that case. This root is bogus.

If $i_D = 1.75 \text{ mA}$, $v_{GS} = V_{DD} - i_D R_S = 8 \text{ V} - 3.85 \text{ V} = 4.15 \text{ V}$. OK, this works.

Finally, $v_{DS} = v_{GS} = 4.15 \text{ V}$.

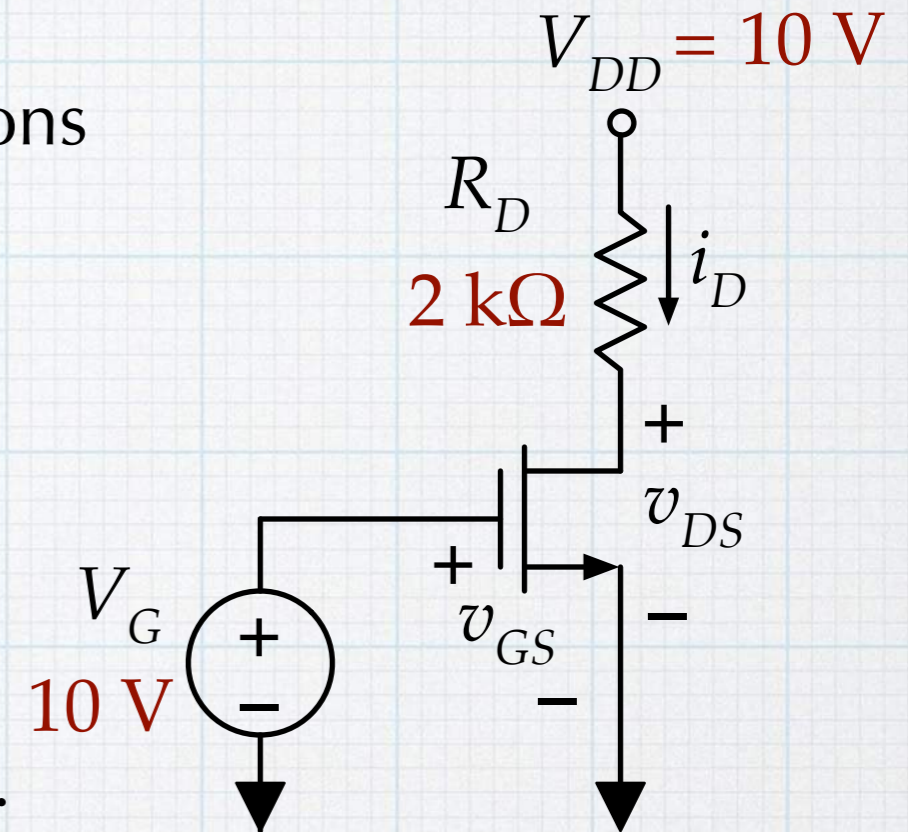
Example 4

For the circuit shown, use the the NMOS equations to find i_D and v_{DS} .

For the NMOS, $V_T = 1.5 \text{ V}$ and $K = 0.5 \text{ mA/V}^2$.

$v_{GS} = V_G = 10 \text{ V} \rightarrow$ the NMOS is on.

This looks like a lot like the first example. So start by assuming that the NMOS is in saturation.



$$i_D = K (v_{GS} - V_T)^2 = (0.5 \text{ mA/V}^2) [10 \text{ V} - 1.5 \text{ V}]^2 = 36.125 \text{ mA}$$

$$v_{DS} = V_{DD} - i_D R_D = 10 \text{ V} - (36.125 \text{ mA}) (2 \text{ k}\Omega) = -62.25 \text{ V}$$

Red Alert! There is a serious problem here. Apparently the NMOS is **not** in saturation. So try the ohmic equation.

$$i_D = K [2 (v_{GS} - V_T) v_{DS} - v_{DS}^2]$$

Unfortunately, we don't know either i_D or v_{DS} . So we need a second equation.

$$i_D = K [2 (v_{GS} - V_T) v_{DS} - v_{DS}^2]$$

Use Ohm's law on the drain resistor to get a second equation:

$$i_{RD} = \frac{V_{DD} - v_{DS}}{R_D} = i_D$$

We can use these to solve for either i_D or v_{DS} . Setting the two equal and solving for v_{DS} is probably slightly easier.

$$\frac{V_{DD} - v_{DS}}{R_D} = K [2 (v_{GS} - V_T) v_{DS} - v_{DS}^2]$$

$$\text{Re-arrange: } v_{DS}^2 - \left[2 (v_{GS} - V_T) + \frac{1}{KR_D} \right] v_{DS} + \frac{V_{DD}}{KR_D} = 0$$

$$\text{Plug in numbers: } v_{DS}^2 - [18 \text{ V}] v_{DS} + 10 \text{ V}^2 = 0$$

$$\text{Solve: } v_{DS} = 0.574 \text{ V or } v_{DS} = 17.43 \text{ V.}$$

It should be obvious that the larger value is way too big – it's bigger than V_{DD} . Also, since the NMOS is in ohmic, we expect v_{DS} to be small. So we choose the smaller value as correct.

$$\text{Lastly: } i_D = \frac{V_{DD} - v_{DS}}{R_D} = \frac{10 \text{ V} - 0.573 \text{ V}}{2\text{k}\Omega} = 4.71 \text{ mA}$$

Example 5

For the circuit shown, use the the NMOS equations to find i_D and v_{DS} .

For the NMOS, $V_T = 1\text{ V}$ and $K = 0.25\text{ mA/V}^2$.

First note that since $i_G = 0$, R_1 and R_2 form a simple voltage divider, and

$$V_G = \frac{R_2}{R_2 + R_1} V_{DD} = 4\text{ V}$$

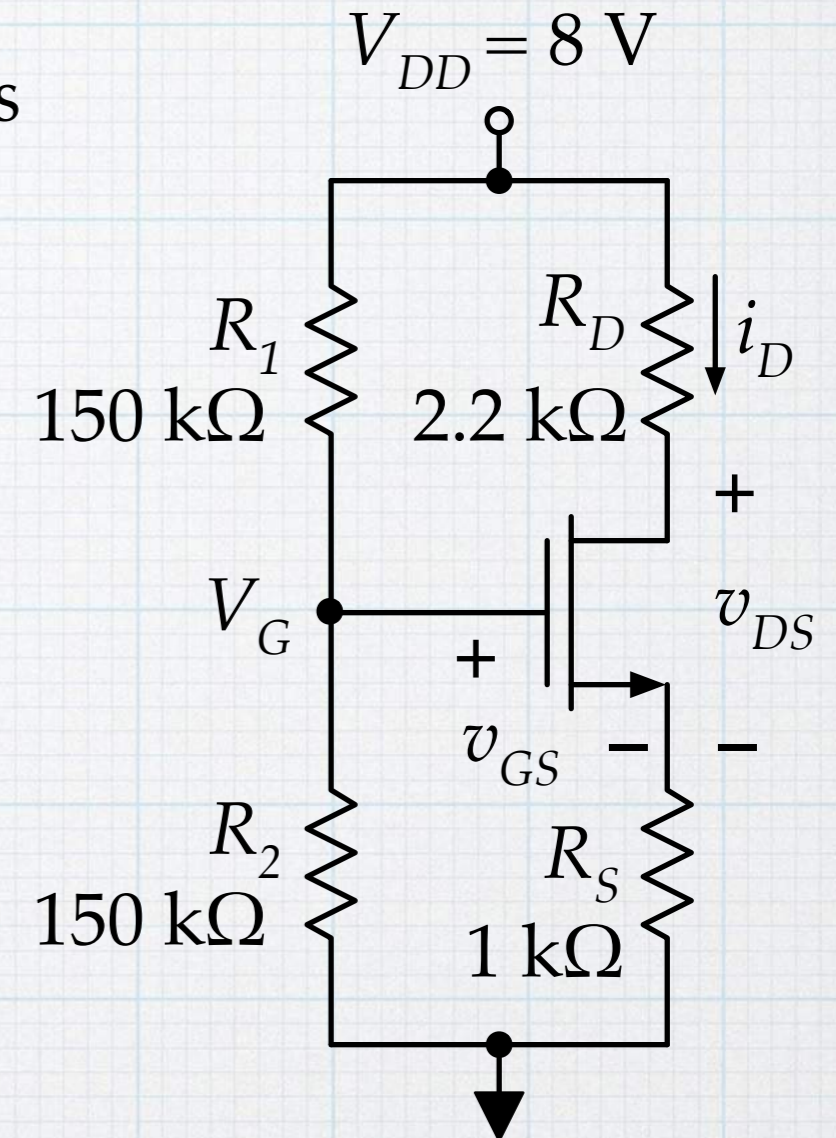
Since $V_G > V_T$, the NMOS should be on. Guess that it is in saturation.

$$v_{GS} = V_G - v_{RS} = V_G - i_D R_S$$

$$i_D = K (v_{GS} - V_T)^2 = K (V_G - i_D R_S - V_T)^2$$

This is exactly the same as example 2.

$$i_D^2 - \left[\frac{1}{KR_S^2} + 2 \left(\frac{V_G - V_T}{R_S} \right) \right] i_D + \left[\frac{V_G - V_T}{R_S} \right]^2 = 0$$



$$i_D^2 - \left[\frac{1}{KR_S^2} + 2 \left(\frac{V_G - V_T}{R_S} \right) \right] i_D + \left[\frac{V_G - V_T}{R_S} \right]^2 = 0$$

Plug in the numbers:

$$i_D^2 - [10 \text{ mA}] i_D + 9 \text{ mA}^2 = 0$$

Use the quadratic equation:

$$i_D = 9 \text{ mA} \text{ or } i_D = 1 \text{ mA}.$$

Which is right? Check the v_{GS} for both.

If $i_D = 9 \text{ mA}$, $v_{GS} = V_G - i_D R_S = 4 \text{ V} - 8.24 \text{ V} = -5 \text{ V}$, and

if $i_D = 1 \text{ mA}$, $v_{GS} = V_G - i_D R_S = 4 \text{ V} - 1 \text{ V} = 3 \text{ V}$.

Clearly, $i_D = 1 \text{ mA}$ is the only answer that makes sense.

Finally, $v_{GS} - V_T = 2 \text{ V}$, and $v_{DS} = V_{DD} - i_D R_D - i_D R_S = 4.8 \text{ V}$.

$v_{DS} > v_{GS} - V_T \rightarrow$ saturation confirmed.

Example 6

Same as example 5, but values for R_2 is increased to 680 k Ω . It is the same NMOS: $V_T = 1$ V and $K = 0.25$ mA/V².

Following the same procedure as Example 5, we obtain $V_G = 6.55$ V. Guessing saturation and performing the same calculation to find i_D ,

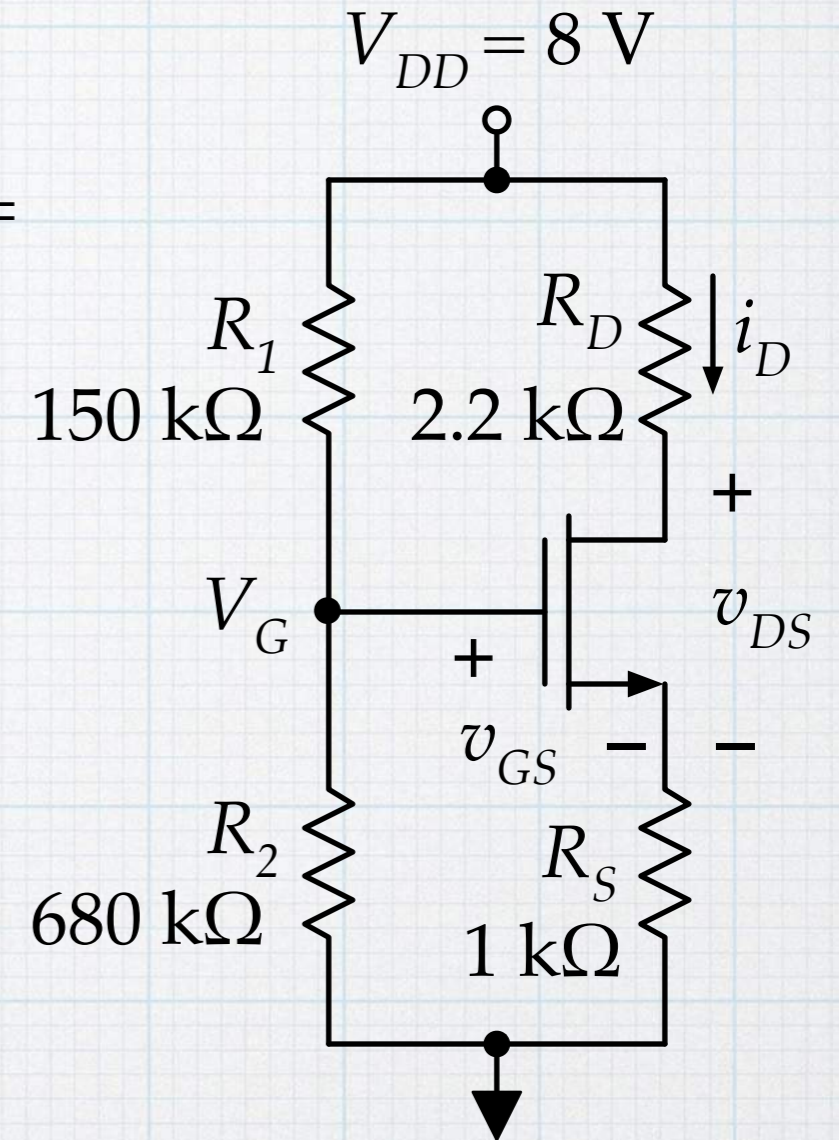
$$i_D = 2.44 \text{ mA or } i_D = 12.7 \text{ mA.}$$

Again, the larger of these is clearly too big to make any sense. Checking the smaller value for consistency with saturation:

$$v_{GS} - V_T = V_G - i_D R_S - V_T = 3.11 \text{ V, and}$$

$$v_{DS} = V_{DD} - i_D R_D - i_D R_S = 0.19 \text{ V.}$$

Oops!! $v_{DS} < v_{GS} - V_T \rightarrow$ This is not in saturation!



So start over, assuming ohmic operation:

$$i_D = K [2 (v_{GS} - V_T) v_{DS} - v_{DS}^2]$$

$$v_{GS} - V_T = V_G - i_D R_S - V_T$$

$$v_{DS} = V_{DD} - i_D R_D - i_D R_S$$

This is gonna be messy...

$$\frac{i_D}{K} = 2 [V_G - i_D R_S - V_T] [V_{DD} - i_D (R_D + R_S)] - [V_{DD} - i_D (R_D + R_S)]^2$$

After a whole lotta algebra...

$$i_D^2 - \left[\frac{1}{K (R_S^2 - R_D^2)} + \frac{2R_S V_{DD}}{(R_S^2 - R_D^2)} + \frac{2(V_G - V_T)}{(R_S - R_D)} - \frac{2V_{DD}}{(R_S - R_D)} \right] i_D$$

(Ouch! That one hurt...)

$$+ \left[\frac{2(V_G - V_T) V_{DD} - V_{DD}^2}{(R_S^2 - R_D^2)} \right] = 0$$

Plug in the numbers: $i_D^2 + [1.12 \text{ mA}] i_D - 6.45 \text{ mA}^2 = 0$

and the two roots are: $i_D = 2.04 \text{ mA}$ and $i_D = \cancel{-3.16 \text{ mA}}$.

Definitely ohmic.

$$v_{DS} = 8 \text{ V} - (2.04 \text{ mA})(2.2 \text{ k}\Omega + 1 \text{ k}\Omega) = 1.47 \text{ V}$$

Example 7

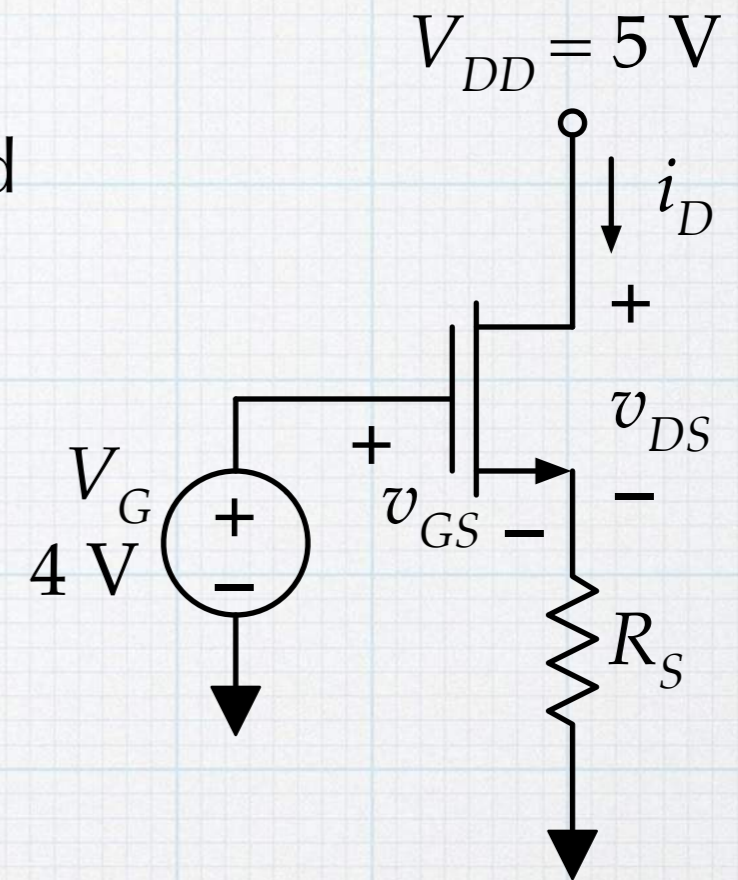
Design the circuit at right (by choosing K for the NMOS and the value of R_S) so that $i_D = 1 \text{ mA}$ and $v_{DS} = 2.5 \text{ V}$. The NMOS has $V_T = 1 \text{ V}$.

By writing a loop equation around the drain-source loop, we see that $v_{RS} = V_{DD} - v_{DS} = 2.5 \text{ V}$. And so $R_S = 2.5 \text{ V} / 1 \text{ mA} = 2.5 \text{ k}\Omega$.

Now writing a loop equation around the gate-source loop, we see that $v_{GS} = V_G - v_{RS} = 1.5 \text{ V}$. This value of v_{GS} means that the NMOS must be operating in saturation.

Then, since in saturation $i_D = K(v_{GS} - V_T)^2$,

$$K = \frac{i_D}{(v_{GS} - V_T)^2} = \frac{1 \text{ mA}}{(1.5 \text{ V} - 1 \text{ V})^2} = 4 \frac{\text{mA}}{\text{V}^2}$$



Example 8

Design the circuit at right (by choosing K for the NMOS and the value of R_D) so that $i_D = 10 \text{ mA}$ and $v_{DS} = 0.2 \text{ V}$. The NMOS has $V_T = 1 \text{ V}$. How much power is being dissipated in the resistor and the NMOS?

If $v_{DS} = 0.2 \text{ V}$, then $v_{RD} = 9.8 \text{ V}$. For a current of 10 mA , $R_D = v_{RD} / i_D = 9.8 \text{ V} / 10 \text{ mA} = 0.98 \text{ k}\Omega$.

With $v_{GS} = 5 \text{ V}$ and $v_{DS} = 0.2 \text{ V}$, the NMOS must be working in the ohmic region. For ohmic operation:

$$i_D = K [2 (v_{GS} - V_T) v_{DS} - v_{DS}^2]$$

$$K = \frac{i_D}{[2 (v_{GS} - V_T) v_{DS} - v_{DS}^2]} = \frac{10 \text{ mA}}{2 (5 \text{ V} - 1 \text{ V}) (0.2 \text{ V}) - (0.2 \text{ V})^2} = 6.41 \frac{\text{mA}}{\text{V}^2}$$

$$P_{RD} = (9.8 \text{ V}) (10 \text{ mA}) = 98 \text{ mW} \quad P_{NMOS} = (0.2 \text{ V}) (10 \text{ mA}) = 2 \text{ mW}$$

