Transistor Hybrid Model

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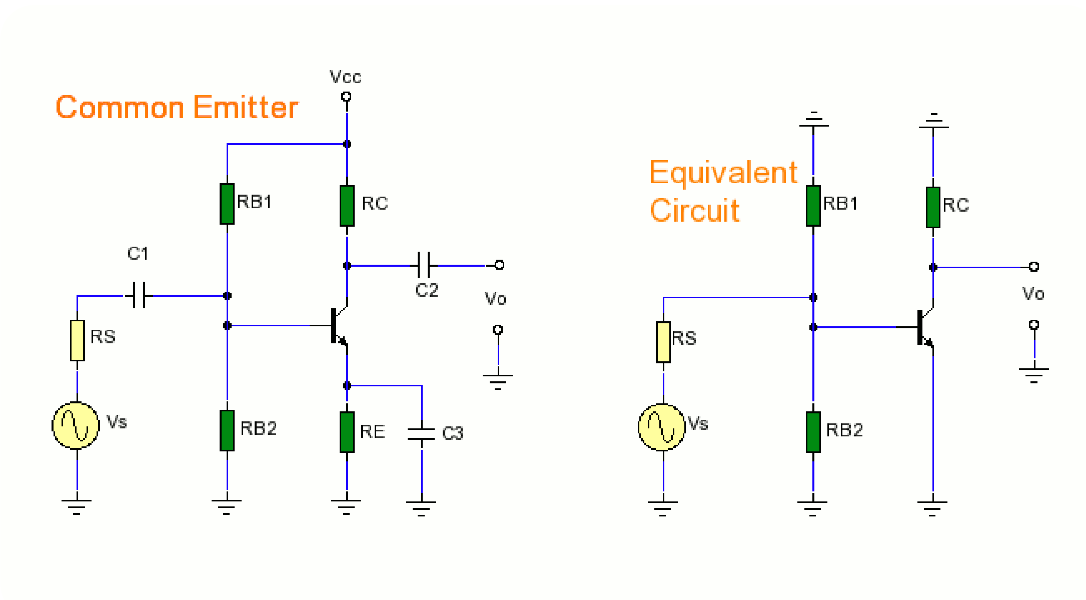
Introduction
The primary function of a "model" is to predict the behaviour of a device in a particular operating region. At dc the bipolar junction transistor (BJT) and some of its biasing techniques have already been described, see these articles:

BJT Biasing
Transistor as a Switch

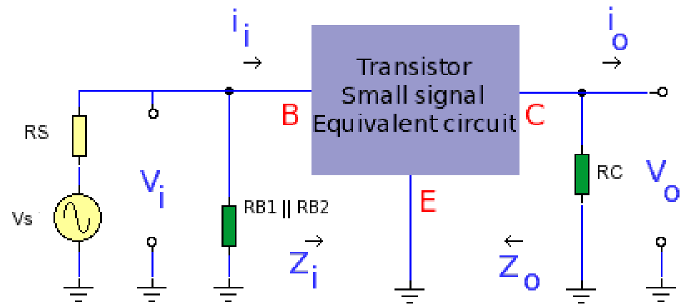
The behaviour of the BJT in the sinusoidal ac domain is quite different from its dc domain. At dc the BJT usually works at in either saturation or cutoff regions. In the ac domain the transistor works in the linear region and effects of capacitance between terminals, input impedance, output conductance, etc all have to be accounted for. The small-signal ac response can be described by two common models: the *hybrid model* and r_e *model*. The models are equivalent circuits (or combination of circuit elements) that allow methods of circuit analysis to predict performance.

Transistor Hybrid Model
To demonstrate the Hybrid transistor model an ac equivalent circuit must be produced. The left hand diagram below is a single common emitter stage for analysis.



At ac the reactance of coupling capacitors C1 and C2 is so low that they are virtual short circuits, as does the bypass capacitor C3. The power supply (which will have filter capacitors) is also a short circuit as far as ac signals are concerned. The equivalent circuit is shown above on the right hand diagram. The input signal generator is shown as Vs and the generators source impedance as Rs.

As RB1 and RB2 are now in parallel the input impedance will be RB1 || RB2. The collector resistor RC also appears from collector to emitter (as emitter is bypassed). See below :



The blue rectangle now represents the small signal ac equivalent circuit and can now start work on the hybrid equivalent circuit.
The hybrid model has four h-parameters. The "h" stands for hybrid because the parameters are a mix of impedance, admittance and dimensionless units. In common emitter the parameters are:

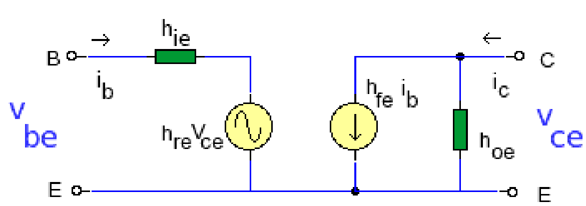
h_{ie} input impedance (Ω)

h_{re} reverse voltage ratio (dimensionless)

h_{fe} forward current transfer ratio (dimensionless)

h_{oe} output admittance (Siemen)

Note that lower case suffixes indicate small signal values and the last suffix indicates the mode so h_{ie} is input impedance in common emitter, h_{fb} would be forward current transfer ration in common base mode, etc. The hybrid model for the BJT in common emitter mode is shown below:



The hybrid model is suitable for small signals at mid band and describes the action of the transistor. Two equations can be derived from the diagram, one for input voltage v_{be} and one for the output i_c:

$$v_{be} = h_{ie}\, i_b + h_{re}\, v_{ce}$$
$$i_c = h_{fe}\, i_b + h_{oe}\, v_{ce}$$

If i_b is held constant (ib=0) then h_{re} and h_{oe} can be solved:

$$h_{re} = v_{be} / v_{ce} \mid i_b = 0$$
$$h_{oe} = i_c / v_{ce} \mid i_b = 0$$

Also if v_{ce} is held constant (v_{ce}=0) then h_{ie} and h_{fe} can be solved:

$$h_{ie} = v_{be} / i_b \mid v_{ce} = 0$$
$$h_{fe} = i_c / i_b \mid v_{ce} = 0$$

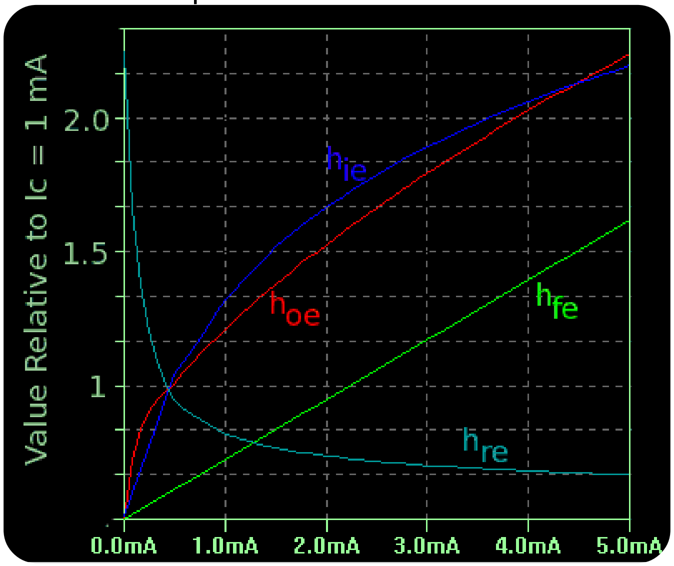
These are the four basic parameters for a BJT in common emitter. Typical values are $h_{re} = 1 \times 10^{-4}$, h_{oe} typical value 20uS, h_{ie} typically 1k to 20k and h_{fe} can be 50 - 750. The H-parameters can often be found on the transistor datasheets. The table below lists the four h-parameters for the BJT in common base and common collector (emitter follower) mode.

h-parameters of Bipolar Junction Transistor

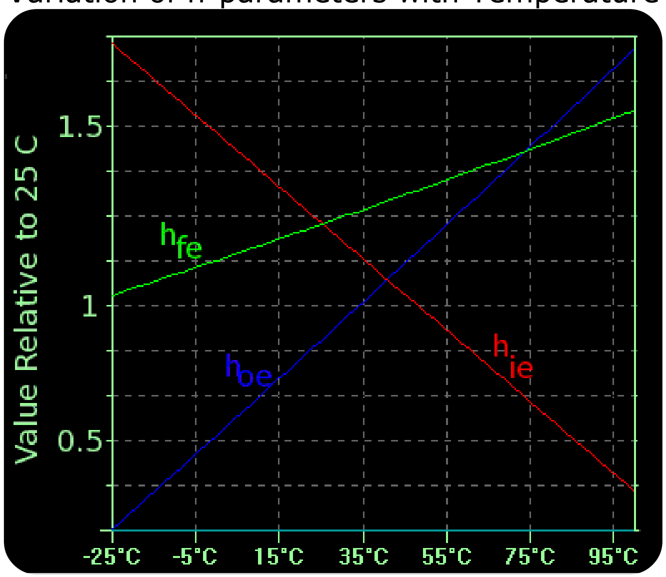
Common Base	Common Emitter	Common Collector	Definitions
$h_{ib} = \frac{v_{eb}}{i_e}$	$h_{ie} = \frac{v_{be}}{i_b}$	$h_{ic} = \frac{v_{bc}}{i_b}$	Input Impedance with Output Short Circuit
$h_{rb} = \frac{v_{eb}}{v_{cb}}$	$h_{re} = \frac{v_{be}}{v_{ce}}$	$h_{rc} = \frac{v_{bc}}{v_{ec}}$	Reverse Voltage Ratio Input Open Circuit
$h_{fb} = \frac{i_c}{i_e}$	$h_{fe} = \frac{i_c}{i_b}$	$h_{fc} = \frac{i_e}{i_b}$	Forward Current Gain Output Short Circuit
$h_{ob} = \frac{i_c}{v_{cb}}$	$h_{oe} = \frac{i_c}{v_{ce}}$	$h_{oc} = \frac{i_e}{v_{ec}}$	Output Admittance Input Open Circuit

H-parameters are not constant and vary with temperature, collector current and collector emitter voltage. For this reason when designing a circuit the hybrid parameters should be measured under the same conditions as the actual circuit. Below are graphs of the variation of h-parameters with temperature and collector current.

Variation of h-parameters with Collector Current



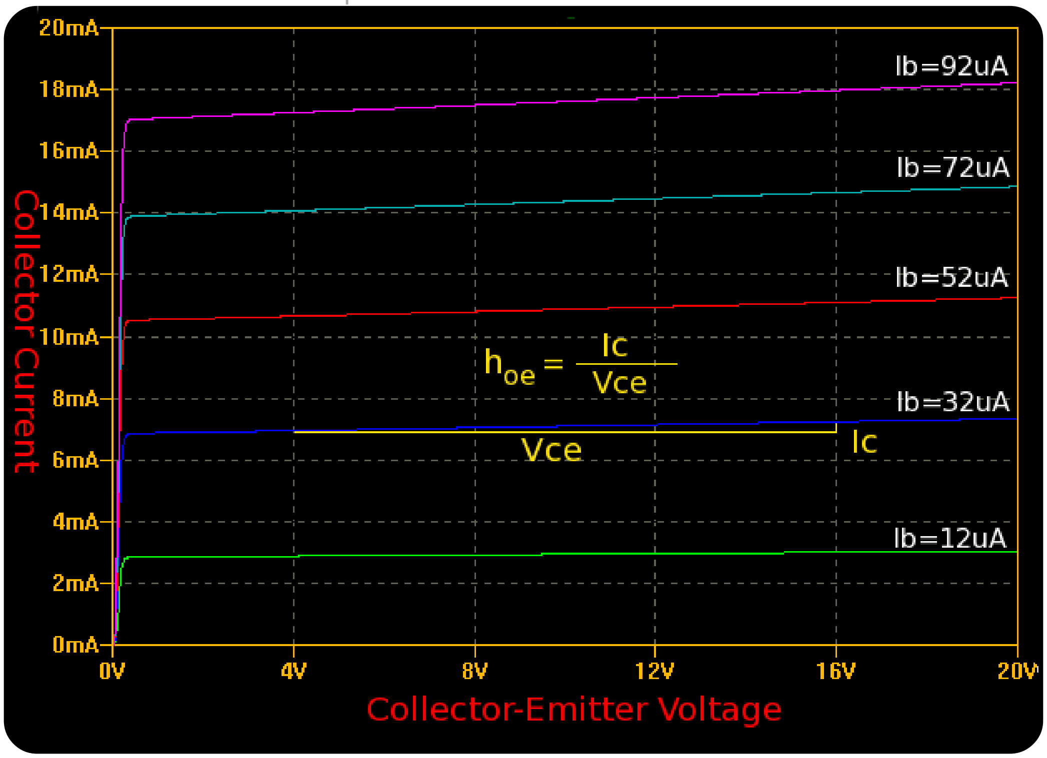
Variation of h-parameters with Temperature



Output Characteristic Curves

The graph below, shows the output characteristic curves for a 2N3904 transistor in common emitter mode. The output curves are quite useful as they show the change in collector current for a range of collector emitter voltages.

Output Characteristics for 2N3904



In addition, because the base currents are also known, then two small signal parameters, h_{fe} and h_{oe} can be determined straight from the graph. The almost flat portion of the curves, shows that the transistor behaves as a constant current generator. However, in saturation the steepness of the curves (between 0 and 0.4 Vce) show a sharp drop in h_{fe}. This is an important fact to consider, if using the transistor as a switch.

Typical h-parameter Values

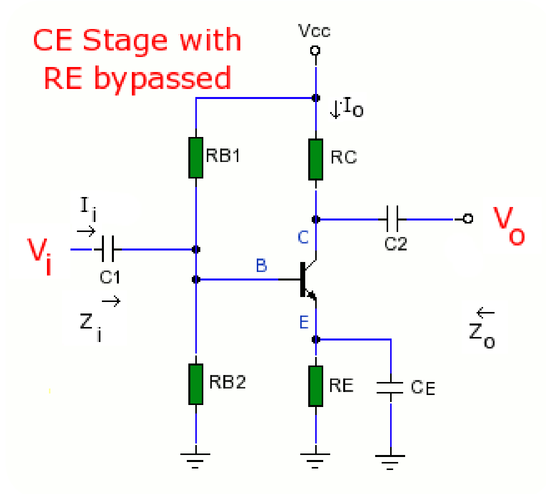
h-parameters are not constant and vary with both temperature and collector current. Typical values for 1mA collector currents are:

$h_{ie} = 1000\ \Omega$ $h_{re} = 3 \times 10^{-4}$ $h_{oe} = 3 \times 10^{-6}S$ $h_{fe} = 250$

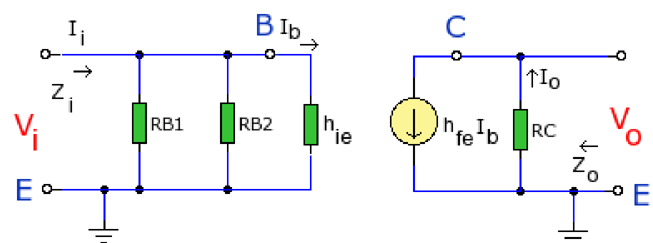
Examples

CE Stage with RE Bypassed

The h-parameter model will be applied to a single common emitter (CE) stage with the emitter resistor (RE) bypassed. The model will be used to build equations for voltage gain, current gain, input and output impedance. The circuit is shown below:



The small signal parameter $h_{re}V_{ce}$ is often too small to be considered so the input resistance is just h_{ie}. Often the output resistance h_{oe} is often large compared wi the the collector resistor RC and its effects can be ignored. The h-parameter equivalent model is now simplified and drawn below:



Input Impedance Z_i

The input impedance is the parallel combination of bias resistors RB1 and RB2. As the power supply is considered short circuit at small signal levels then RB1 and RB2 are in parallel. RBB will represent the parallel combination:

$$R_{BB} = RB1 \;||\; RB2 = \frac{RB1\ RB2}{RB1 + RB2}$$

As R_{BB} is in parallel with h_{ie} then:

$$Z_i = R_{BB} \;||\; h_{ie}$$

Output Impedance Z_o

As $h_{fe}I_b$ is an ideal current generator with infinite output impedance, then output impedance looking into the circuit is:

$$Z_o = RC$$

Voltage Gain A_v

Note the − sign in the equation, this indicates phase inversion of the output waveform.

$$V_o = -I_o\, RC = -h_{fe}\, I_b\, RC$$

as $I_b = V_i / h_{ie}$ then:

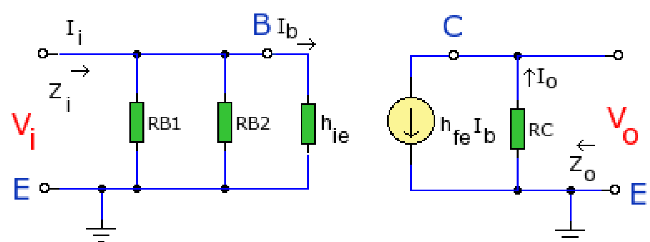
$$= -h_{fe} \frac{V_i}{h_{ie}} RC$$

$$= \frac{-h_{fe}}{h_{ie}} RC\, V_i$$

$$A_v = \frac{V_o}{V_i} = \frac{-h_{fe}\, RC}{h_{ie}}$$

Current Gain A_i

The current gain is the ratio Io / Ii. At the input the current is split between the parallel branch R_{BB} and h_{ie}. So looking at the equivalent h-parameter model again (shown below):



The current divider rule can be used for I_b:

$$I_b = \frac{R_{BB}\, I_i}{R_{BB} + h_{ie}}$$

$$\frac{I_b}{I_i} = \frac{R_{BB}}{R_{BB} + h_{ie}}$$

At the output side, $I_o = h_{fe}\, I_b$

re-arranging $I_o / I_b = h_{fe}$

$$A_i = \frac{I_o}{I_i} = \frac{I_o\, I_b}{I_b\, I_i} = h_{fe} \frac{R_{BB}}{R_{BB} + h_{ie}}$$

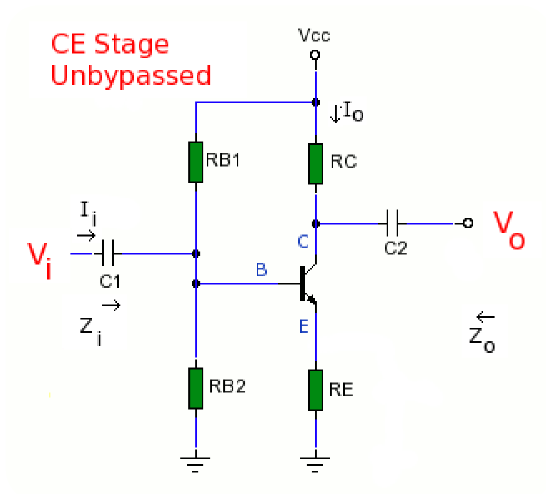
$$A_i = \frac{R_{BB}\, h_{fe}}{R_{BB} + h_{ie}}$$

If RBB >> hie then,

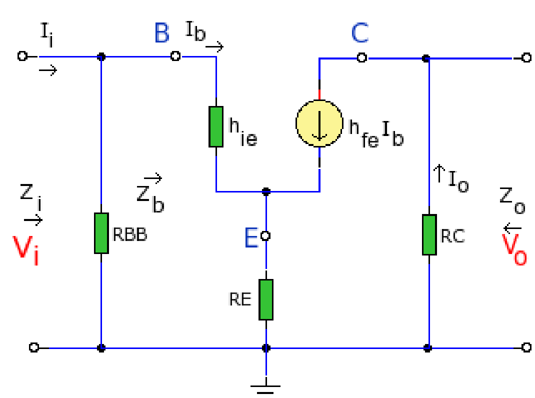
$$A_i \approx \frac{R_{BB}\, h_{fe}}{R_{BB}} = h_{fe}$$

CE Stage with RE Unbypassed

The h-parameter model of a common emitter stage with the emitter resistor unbypassed is now shown. The model will be used to build equations for voltage gain, current gain, input and output impedance. The circuit is shown below:



As in the previous example, RB1 and RB2 are in parallel, the bias resistors are replaced by resistance R_{BB}, but as RE is now unbypassed this resistor appears in series with the emitter terminal. The hybrid small signal model is shown below, once again effects of small signal parameters $h_{re}V_{ce}$ and h_{oe} have been omitted.



Input Impedance Z_i

The input impedance Z_i is the bias resistors R_{BB} in parallel with the impedance of the base, Z_b.

$$Z_b = h_{ie} + (1 + h_{fe})\ RE$$

Since hfe is normally much larger than 1, the equation can be reduced to:

$$Z_b = h_{ie} + h_{fe}\ RE$$

$$Z_i = R_{BB} \,||\, (h_{ie} + h_{fe}\ RE)$$

Output Impedance Z_o

With Vi set to zero, then Ib = 0 and $h_{fe}I_b$ can be replaced by an open-circuit. The output impedance is:

$$Z_o = RC$$

Voltage Gain A_v

Note the − sign in the equation, this indicates phase inversion of the output waveform.

$$I_b = \frac{V_i}{Z_b}$$

$$V_o = -I_o\ RC = -h_{fe}\ I_b\ RC$$

$$= -h_{fe} \frac{V_i}{Z_b} RC$$

$$A_v = \frac{V_o}{V_i} = \frac{-h_{fe}\ RC}{Z_b}$$

As $Z_b = h_{ie} + h_{fe}$ RE often the product h_{fe}RE is much larger than hie, so Z_b can reduced to the approximation:

$$Z_b \approx h_{fe}RE$$

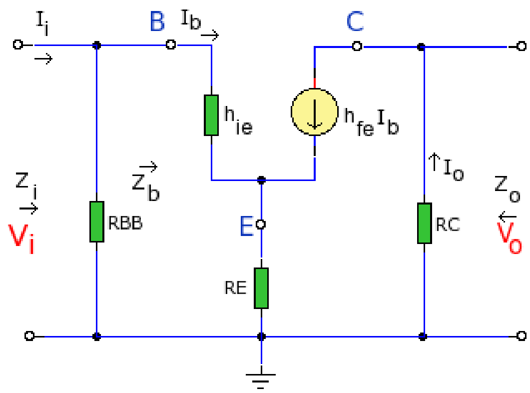
$$\therefore A_v = \frac{-h_{fe}RC}{h_{fe}RE}$$

$$A_v = \frac{V_o}{V_i} = -\frac{RC}{RE}$$

Current Gain A_i

The current gain is the ratio Io / Ii. At the input the current is split between the parallel branch R_{BB} and Z_b. So looking at the equivalent h-parameter model again (shown below):

CE Stage RE Unbypassed



The current divider rule can be used for I_b:

$$I_b = \frac{R_{BB}\, I_i}{R_{BB} + Z_b}$$

$$\frac{I_b}{I_i} = \frac{R_{BB}}{R_{BB} + Z_b}$$

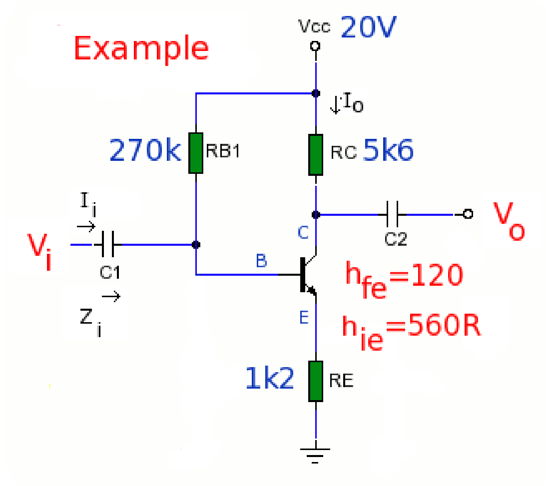
At the output side, $I_o = h_{fe}\, I_b$

re-arranging $I_o / I_b = h_{fe}$

$$A_i = \frac{I_o}{I_i} = \frac{I_o\, I_b}{I_b\, I_i} = h_{fe}\frac{R_{BB}}{R_{BB} + Z_b}$$

$$A_i = \frac{R_{BB}\, h_{fe}}{R_{BB} + Z_b}$$

Example CE Stage



The hybrid parameters must be known to use the hybrid model, either from the datasheet or measured. In the above circuit, Zi, Zo, Av, and Ai will now be calculated. Note that this CE stage uses a single bias resistor RB1 which is the value RBB.

Z_i

$$Z_b = h_{ie} + (1 + h_{fe})\ RE$$

$$= 0.56k + (\ 1 + 120)\ 1.2k = 145.76k$$

$$Z_i = RB \mid\mid Z_b$$

$$\boxed{Z_i = 270k \mid\mid 145.76k = 94.66k}$$

Z_o

$$\boxed{Z_o \approx 5.6k}$$

A_v

$$A_v = -\frac{h_{fe}\ RC}{Z_b}$$

$$= -\frac{120 \times 5.6k}{145.76k}$$

$$\boxed{A_v = -\ 4.61}$$

A_i

$$A_i = \frac{R_{BB}\ h_{fe}}{R_{BB} + Z_b}$$

$$= \frac{270k \times 120}{}$$

270k + 145.76k

$A_i = 77.93$

<u>Summary</u>

The hybrid model is limited to a particular set of operating conditions for accuracy. If the device is operated at a different collector current, temperature or Vce level from the manufacturers datasheet then the h parameters will have to be measured for these new conditions. The hybrid model has parameters for output impedance and reverse voltage ratio which can be important in some circuits.

Return to Circuit Theory