Chapter Four: Network Theorems

4.1 Introduction

This chapter will introduce the important fundamental theorems of network analysis. Included are the superposition and Thévenin theorems.

4.2 Superposition Theorem

The superposition theorem is a method which allows us to determine the current through or the voltage across any resistor or branch in a network.

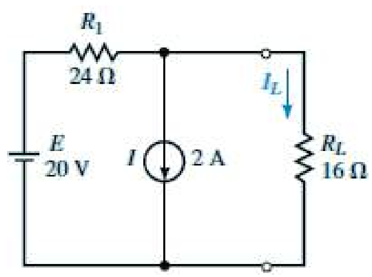
The theorem states the following:
The total current through or voltage across a resistor or branch may be determined by summing the effects due to each independent source.

Steps to apply superposition principles:

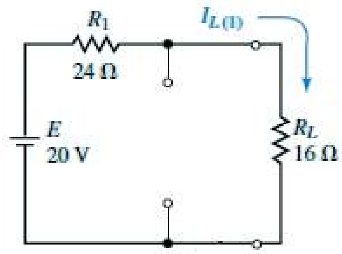
1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources

Note: removing the sources means a voltage source is replaced by short circuit, while a current source is replaced by open circuit.

Example 4.1:
For the circuit shown below, use superposition method to find the current in the load resistor, R_L

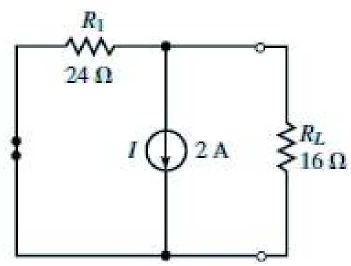


Solution:
Consider the effects of a 20V source, as shown in the figure below.



$$I_{L(1)} = \frac{20V}{24\Omega + 16\Omega} = 0.5A$$

Consider the effects of a 2A source.

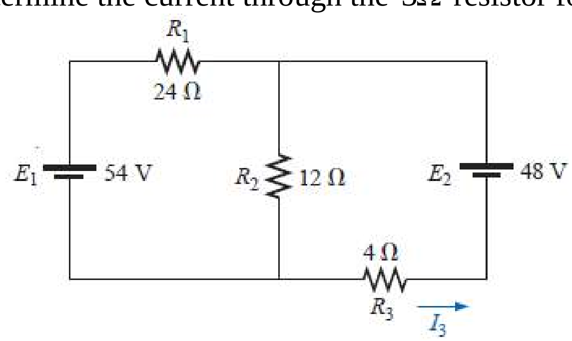


$$I_{L(2)} = \frac{2A * 24\Omega}{24\Omega + 16\Omega} = 1.2A$$

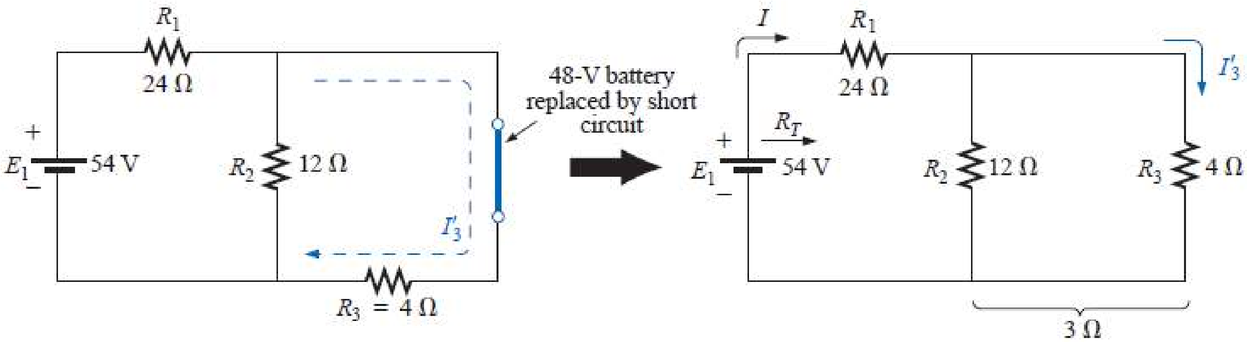
$$I_L = I_{L(1)} - I_{L(2)} = 0.5 - 1. = -0.7A$$

Example 4.2:

Using superposition to determine the current through the 3Ω resistor for the circuit shown below



Solution:
Consider the effects of a 54 V source, as shown below



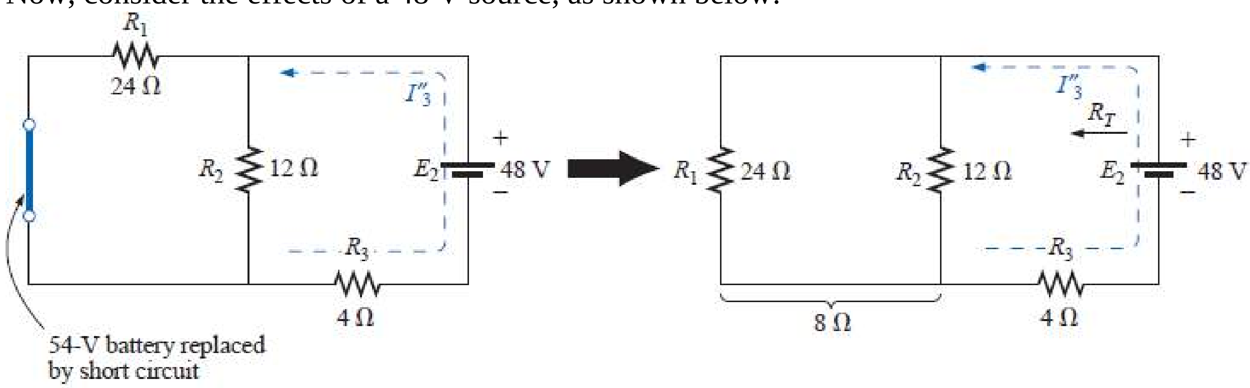
$R_T = R_1 + R_2 // R_3 = 24 + 3 = 27\Omega$

$I = \frac{E_1}{R_T} = \frac{54}{27} = 2A$

Using current divider rule:

$I_3^{'} = \frac{I * R_2}{R_2 + R_3} = \frac{2*12}{16} = 1.5A$

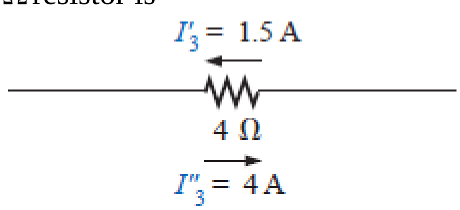
Now, consider the effects of a 48 V source, as shown below:



$R_T = R_3 + R_1 // R_2 = 4 + 8 = 12\Omega$

$I_3^{''} = \frac{E_2}{R_T} = \frac{48}{12} = 4A$

The total current through the 4Ω resistor is



$I_3 = I_3^{''} - I_3^{'} = 4 - 1.5 = 2.5A$

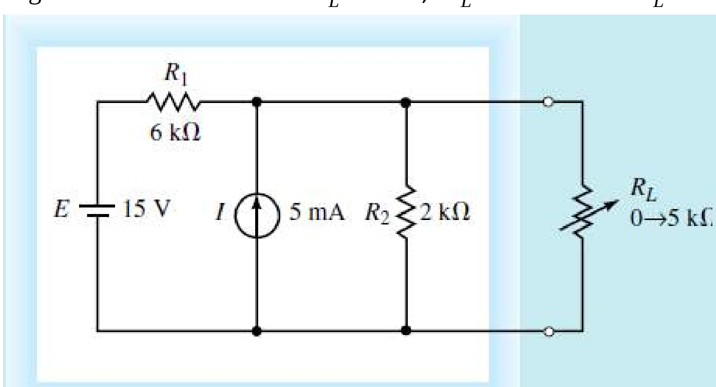
4.3 Thevenin's Theorem

Thévenin's theorem is a circuit analysis technique which reduces any linear bilateral network to an equivalent circuit having only one voltage source and one series resistor.

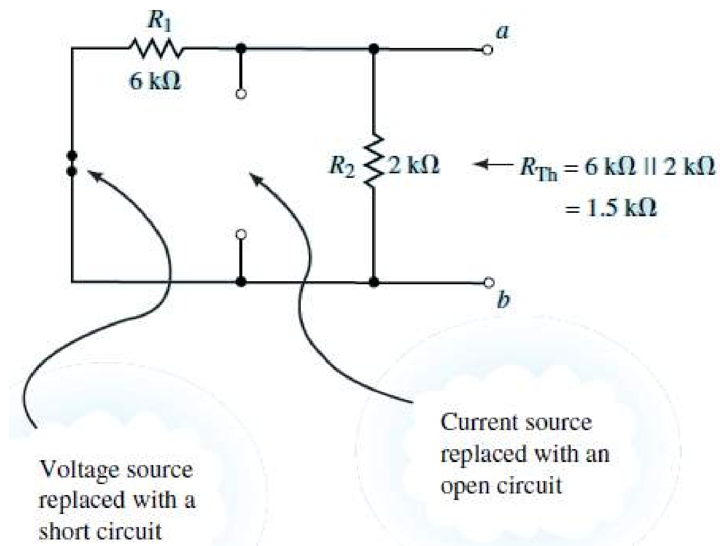
The following steps provide a technique which converts any circuit into its Thévenin equivalent:

1. Remove the load from the circuit.
2. Label the resulting two terminals. We will label them as *a* and *b.*
3. Set all sources in the circuit to zero.
Voltage sources are set to zero by replacing them with short circuits (zero volts).
Current sources are set to zero by replacing them with open circuits (zero amps).
4. Determine the Thévenin equivalent resistance, *R*Th, by calculating the resistance "seen" between terminals *a* and *b*.
5. Replace the sources removed in Step 3, and determine the open-circuit voltage between the terminals.
6. Draw the Thévenin equivalent circuit using the resistance determined in Step 4 and the voltage calculated in Step 5. As part of the resulting circuit, include that portion of the network removed in Step 1

Example 4.3:
Find the Thévenin equivalent circuit of the indicated area in the figure below. Using the equivalent circuit, determine the current through the load resistor when $R_L = 0\Omega$, $R_L = 2k\Omega$ and $R_L = 5k\Omega$



Solution:
Step 1, 2 and 3: after removing the load, labeling the terminals, and setting the sourses to zero, we have the circuit shown below:

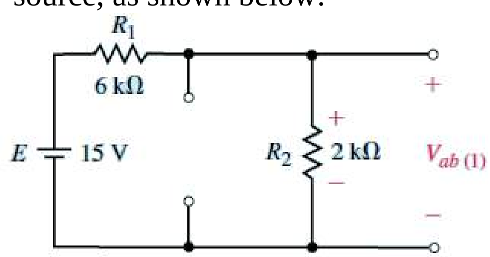


Step 4: the Thevenin resistance of the circuit is

$$R_{Th} = 6k\Omega // 2k\Omega = 1.5k\Omega$$

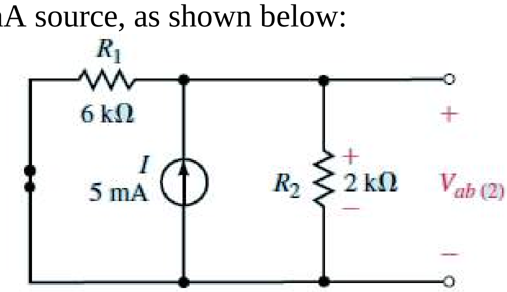
Step 5: we will use the superposition theorem to find the open-circuit voltage V_{ab}

Consider the effects of a 15 V source, as shown below:



$$V_{ab(1)} = \frac{15 * 2k\Omega}{6k\Omega + 2k\Omega} = 3.75 \text{ V}$$

Consider the effects of a 5 mA source, as shown below:

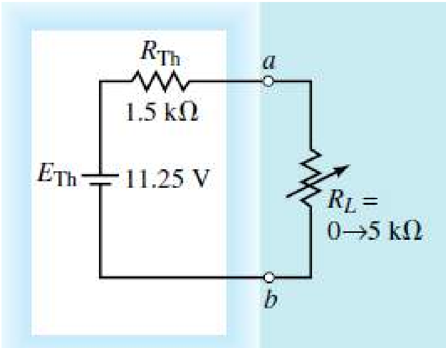


$$V_{ab(2)} = \left(\frac{5mA * 6k\Omega}{8k\Omega}\right) * 2k\Omega = 7.5$$

The Thevenin equivalent voltage is:

$E_{Th} = V_{ab(1)} + V_{ab(2)} = 3.75 + 7.5 = 11.25$ V

Step 6: the resulting Thevenin equivalent circuit is shown in the figure below:



From the circuit above:

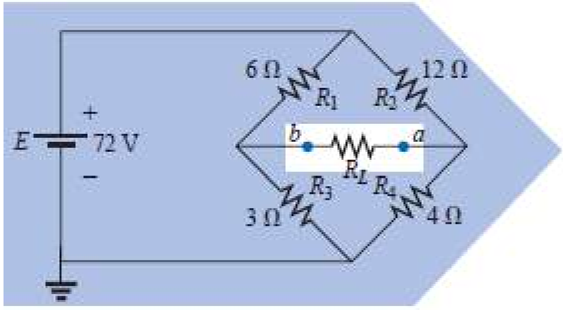
$$R_L = 0\Omega \rightarrow \quad I_L = \frac{11.25V}{1.5k\Omega} = 7.5mA$$

$$R_L = 2k\Omega \rightarrow \quad I_L = \frac{11.25V}{1.5k\Omega + 2k\Omega} = 3.21mA$$

$$R_L = 5k\Omega \rightarrow \quad I_L = \frac{11.25V}{1.5k\Omega + 5k\Omega} = 1.73mA$$

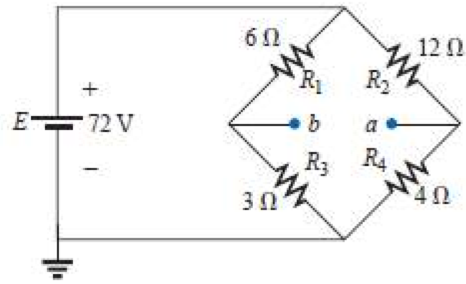
Example 4.4:

Find the Thevenin equivalent circuit for the network n the shaded area of the bridge network shown below:

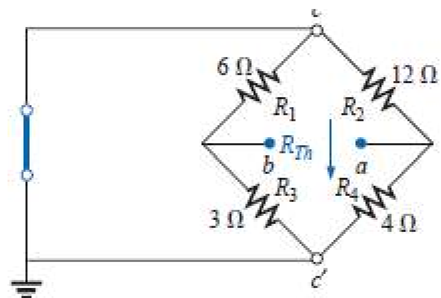


Solution:

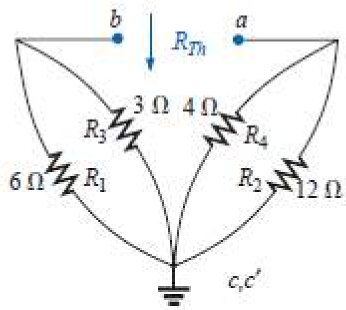
Step 1 and 2 are shown in the figure below



Step 3 is shown below

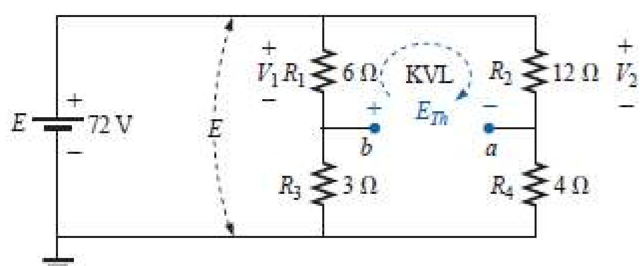


Step 4 is shown below:



$R_{Th} = R_{ab} = 3\Omega // 6\Omega + 4\Omega // 12\Omega = 2 + 3 = 5\Omega$

Step 5:



Applying KVL

$E_{Th} + V_1 - V_2 = 0$

$$V_1 = \frac{E * R_1}{R_1 + R_3} = \frac{72*6}{6+3} = 48\text{V}$$

$$V_2 = \frac{E * R_2}{R_2 + R_4} = \frac{72*12}{12+4} = 54\text{V}$$

$$\therefore E_{Th} = V_2 - V_1 = 54 - 48 = 6\text{V}$$

The Thevenin equivalent circuit is shown below: