$$x_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 2 & -2 \\ 1 & 5 \end{vmatrix}}{-4} = \frac{10+2}{-4} = \frac{12}{-4} = -3$$

Example 3.2:

Solve the following equations

$$x_1 - 2x_3 = -1$$
$$3x_2 + x_3 = 2$$
$$x_1 + 2x_2 + 3x_3 = 0$$

Solution:

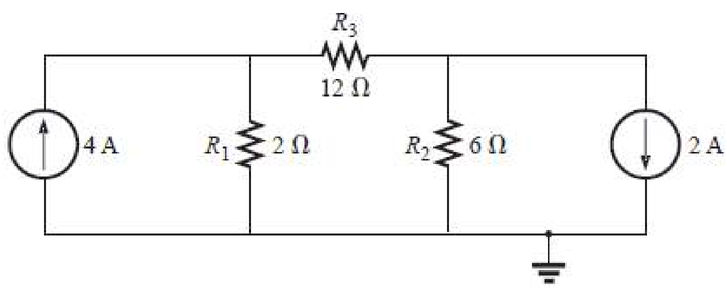
3.2 Node voltage method

Nodal analysis applies KCL to find unknown voltages in a given circuit.

The steps used in solving a circuit using **nodal method** are as follows:
1. Assign a reference node within the circuit and indicate this node as **ground**, assign voltages $(V_1, V_2 ....V_n)$ to the remaining nodes in the circuit
2. Apply Kirchhoff's current law at each node except the reference. Assume all unknown currents leave the node for all each application of Kirchhoff's current law.
3. Solve the resulting equations for the nodal voltages.

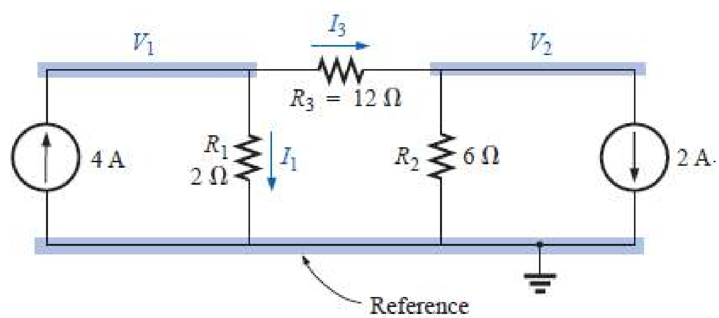
Example 3.3:

Determine the nodal voltages for the circuit shown below:



Solution:

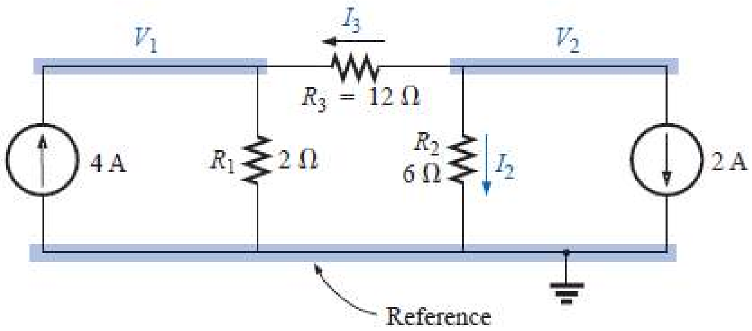
Step 1: as indicate in the figure below



Step 2: applying KCL

For node V_1 : $4 = \frac{V_1}{2} + \frac{V_1 - V_2}{12} \quad \rightarrow \quad 4 = V_1 \left(\frac{1}{2} + \frac{1}{12}\right) - \frac{V_2}{12} \quad \rightarrow \quad 48 = 7V_1 - V_2$ .... (1)

For node V_2 the currents are defined as in the figure below:



$$2+\frac{V_2}{6}=\left(\frac{V_1-V_2}{12}\right) \quad\rightarrow\quad 2=\left(\frac{V_1}{12}\right)-V_2\left(\frac{1}{6}+\frac{1}{12}\right) \quad\rightarrow\quad 24=V_1-3V_2 \quad \text{.............. (2)}$$

Step 3:

$$V_1=\frac{\begin{vmatrix}48 & -1\\ 24 & -3\end{vmatrix}}{\begin{vmatrix}7 & -1\\ 1 & -3\end{vmatrix}}=\frac{-124+24}{-21+1}=\frac{-120}{-20}=6v$$

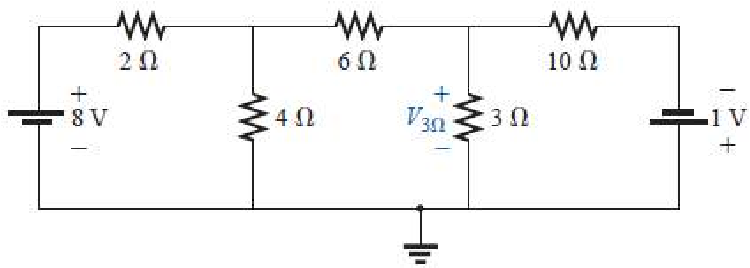
$$V_2=\frac{\begin{vmatrix}7 & 48\\ 1 & 24\end{vmatrix}}{\begin{vmatrix}7 & -1\\ 1 & -3\end{vmatrix}}=\frac{168-48}{-20}=\frac{120}{-20}=-6v$$

3.2.1 Nodal analysis with voltage source

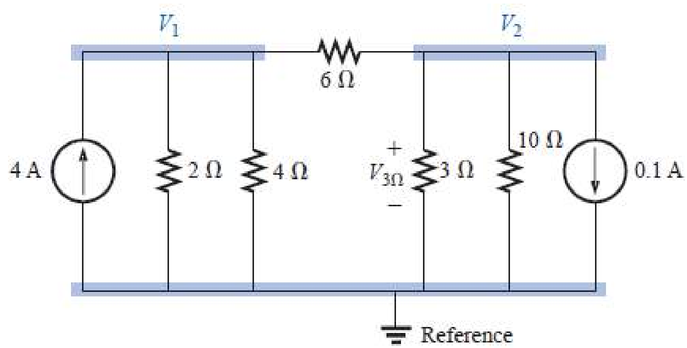
In nodal analysis, a voltage source in series with an element can be transformed into a current source with a shunt element by employing the source transformation.

Example 3.4:

Find the voltage across the 3Ω resistor of the circuit shown below



Solution:



For node V_1 : $4 = \frac{V_1}{2} + \frac{V_1}{4} + \frac{V_1 - V_2}{6} \quad \rightarrow \quad 4 = V_1\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right) - \frac{V_2}{6} \quad \rightarrow$

$$48 = 11V_1 - 2V_2 \quad \text{..........} \quad (1)$$

For node V_2 : $\frac{V_2 - V_1}{6} + V_2\left(\frac{1}{3} + \frac{1}{10}\right) + 0.1 = 0 \quad \rightarrow \quad -\frac{V_1}{6} + V_2\left(\frac{1}{6} + \frac{1}{3} + \frac{1}{10}\right) = -0.1 \quad \rightarrow$

$-\frac{1}{6}V_1 + \frac{3}{5}V_2 = -0.1 \quad \rightarrow \quad -5V_1 + 18V_2 = -3$ .... (2)

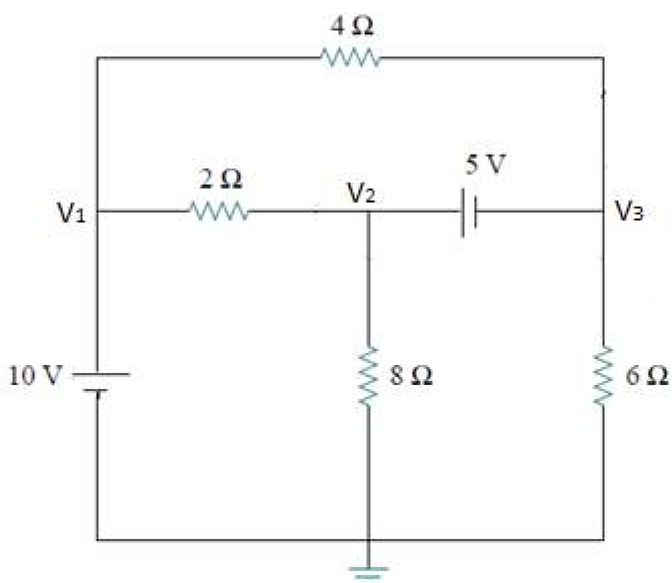
$$V_2 = V_{3\Omega} = \frac{\begin{vmatrix} 11 & 48 \\ -5 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & -2 \\ -5 & 18 \end{vmatrix}} = \frac{-33+240}{198-10} = \frac{207}{188} = 1.101v$$

3.2.1.1. Supernode:

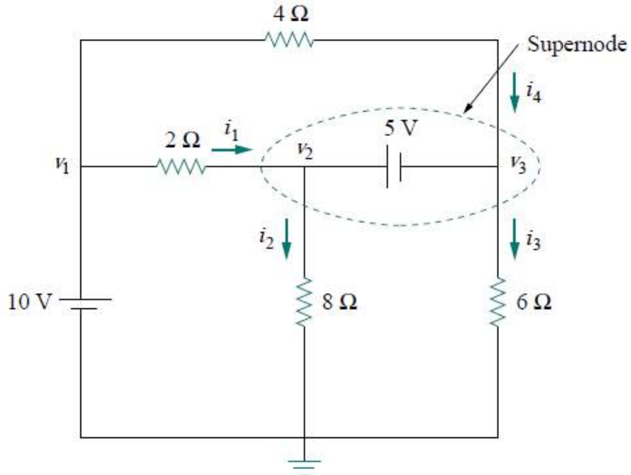
If the voltage source (dependent or independent) is connected between two nonreference nodes, the two non-reference nodes form a *generalized node* or *supernode*; we apply both KCL and KVL to determine the node voltages.

Example 3.5:

Determine the nodal voltages for the circuit shown below:



Solution:



Applying KCL

$I_1 + I_4 = I_2 + I_3$

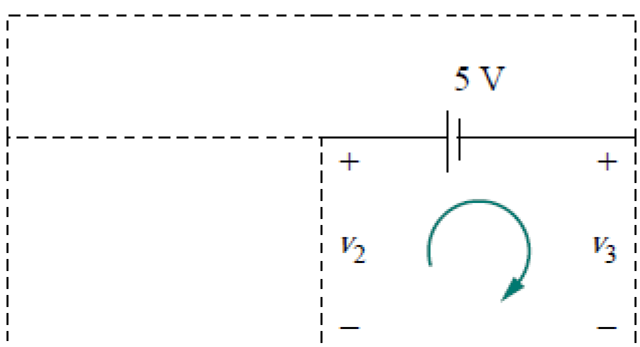
$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2}{8} + \frac{V_3}{6}$$

But $V_1 = 10v$

$$\frac{10 - V_2}{2} + \frac{10 - V_3}{4} = \frac{V_2}{8} + \frac{V_3}{6}$$

$36 = 3V_2 + 2V_3$ .... (1)

To apply KVL to the supernode in the circuit above, we redraw the circuit as shown below



We have

$V_2 - 5 - V_3 = 0 \rightarrow V_2 - V_3 = 5$ .... (2)

Solving for V_2 and V_3 $\rightarrow$ $V_2 = 9.2v$ and $V_3 = 4.2v$

3.3 Mesh (Loop) current method:

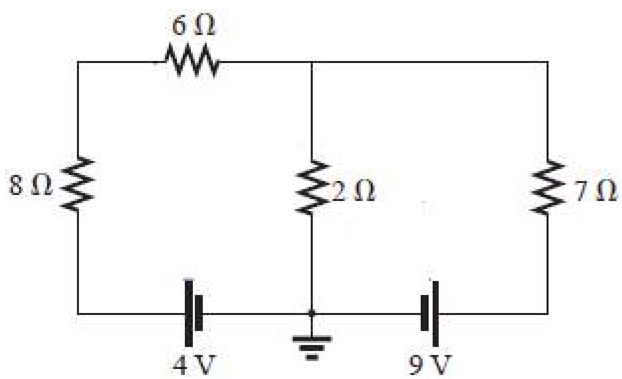
Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Mesh analysis applies KVL to find unknown currents in a given circuit.

Steps to determine mesh currents:

1. Assign mesh currents I_1 , I_2 .... I_n to the *n* meshes.
2. Apply KVL to each of the *n* meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting *n* simultaneous equations to get the mesh currents.

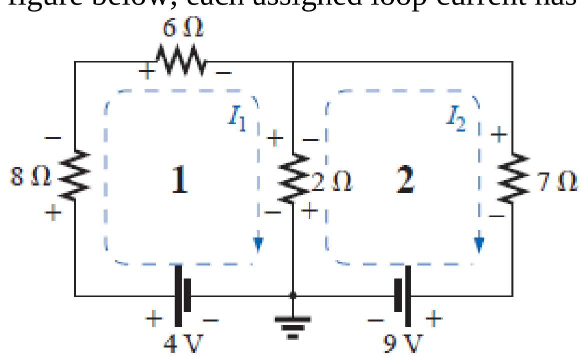
Example 3.6:

Use mesh method to find the current through the 7Ω resistor.



Solution:

Step 1: as indicated in the figure below, each assigned loop current has a clockwise direction.



Step 2: applying KVL

Loop 1: $4-8I_1-6I_1-2(I_1-I_2)=0 \;\rightarrow$

$$4=I_1(8+6+2)-2I_2 \;\rightarrow\; 4=16I_1-2I_2 \qquad \text{............................ (1)}$$

Loop2: $-9-2(I_2-I_1)-7I_2=0 \qquad \rightarrow$

$$-9 = -2I_1 + (2+7)I_2 \quad \rightarrow -9 = -2I_1 + 9I_2$$ .... (2)

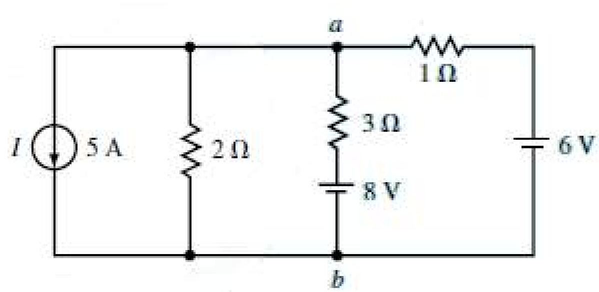
Step 3:

$$I_2 = I_{7\Omega} = \frac{\begin{vmatrix} 16 & 4 \\ -2 & -9 \end{vmatrix}}{\begin{vmatrix} 16 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{-144+8}{144-4} = \frac{-136}{140} = -0.971A$$

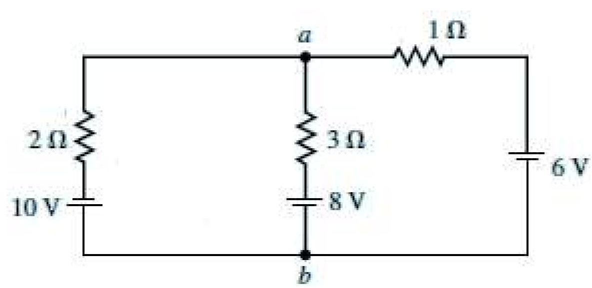
3.3.1 Mesh analysis with current source

In mesh analysis, a current source in parallel with an element can be transformed into a voltage source with a series element by employing the source transformation. This will reduce the number of meshes in the circuit by one.

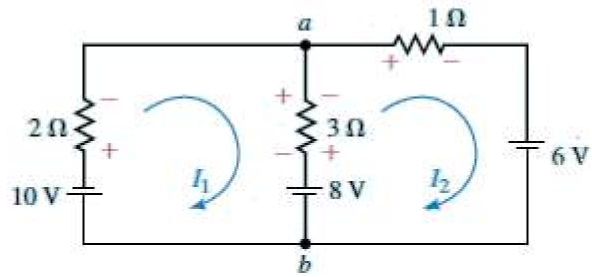
Example 3.7:
Find the current through the 3Ω resistor using mesh analysis method for the circuit shown below:



Solution:
Convert the current source into an equivalent voltage source. The equivalent circuit is shown below:



Step 1:



Step 2:

Loop 1: $-10-2I_1-3(I_1-I_2)-8=0 \quad \rightarrow \quad -18=5I_1-3I_2$ .... (1)

Loop 2: $8-3(I_2-I_1)-1I_2-6=0 \quad \rightarrow \quad 2=-3I_1+4I_2$ .... (2)

Step 3:

$$I_1=\frac{\begin{vmatrix}-18 & -3\\ 2 & 4\end{vmatrix}}{\begin{vmatrix}5 & -3\\ -3 & 4\end{vmatrix}}=\frac{-66}{11}=-6A$$

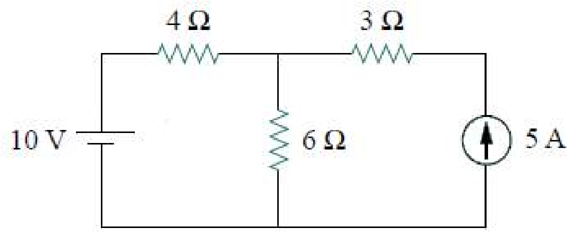
$$I_2=\frac{\begin{vmatrix}5 & -18\\ -3 & 2\end{vmatrix}}{\begin{vmatrix}5 & -3\\ -3 & 4\end{vmatrix}}=\frac{10-54}{11}=\frac{-44}{11}=-4A$$

If the assumed direction of current in the 3Ω resistor is taken to be I_2, then

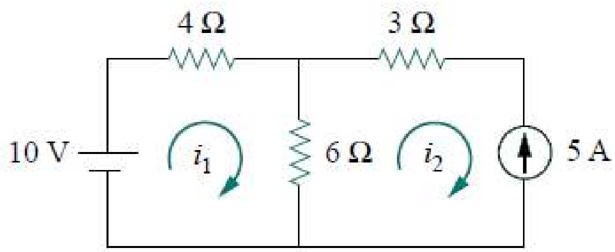
$I=I_2-I_1=-4-(-6)=2A$

The direction of the resultant current is the same as I_2 (upward)

Example 3.8:
Use mesh analysis to find the current in each branch for the circuit shown below



Solution:



Loop 1:

$$10-4I_1-6(I_1-I_2)=0 \quad \rightarrow \quad 10=10I_1-6I_2$$

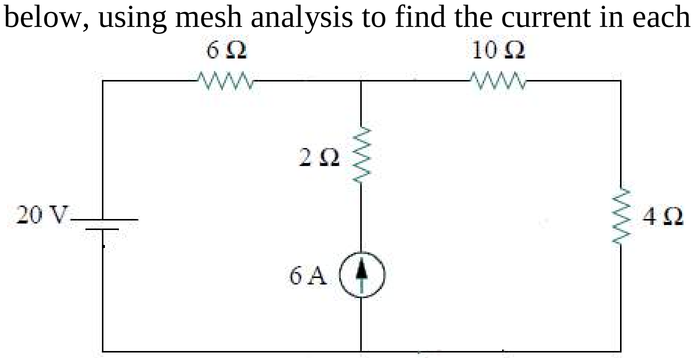
But $I_2=-5A$

$$10=10I_1+30 \rightarrow I_1=\frac{-20}{10}=-2A$$

3.3.1.1 Supermesh currents

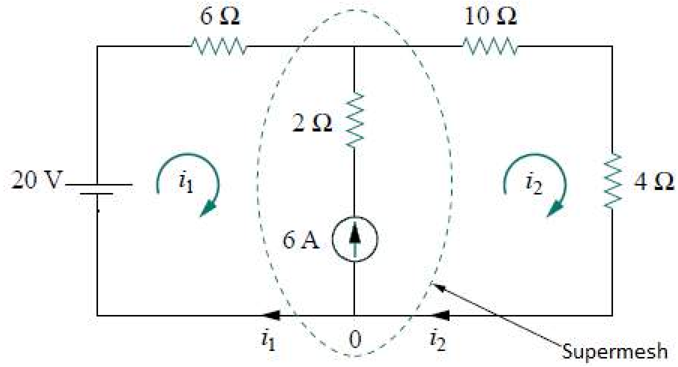
A supermesh results when two meshes have a current source in common. A supermesh requires the application of both KVL and KCL.

Example 3.9:
For the circuit shown below, using mesh analysis to find the current in each branch



Solution:

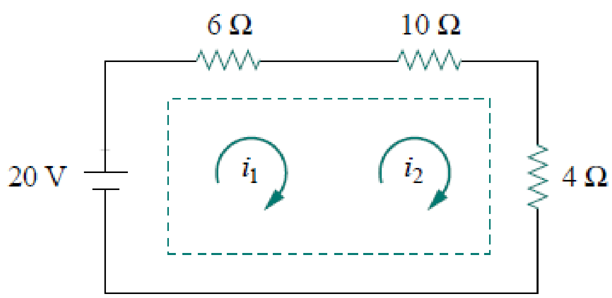
In this circuit, we have supermesh.



Applying KCL:

$$I_1 + 6 = I_2 \quad \text{.................. (1)}$$

To apply KVL to the supermesh, see the equivalent circuit



$$20 - 6I_1 - 10I_2 - 4I_2 = 0 \;\rightarrow\; 20 = 6I_1 + 14I_2 \quad \text{................... (2)}$$

From equation (1) and (2):

$I_1 = -3.2A$

$I_2 = 2.8A$