Chapter two: Circuit Transformations

2.1 Series Circuits

A series circuit is constructed by combining various elements in series, as shown in figure 2.1. Two elements are in series if they have only one point in common that is not connected to other current carrying elements of the network.

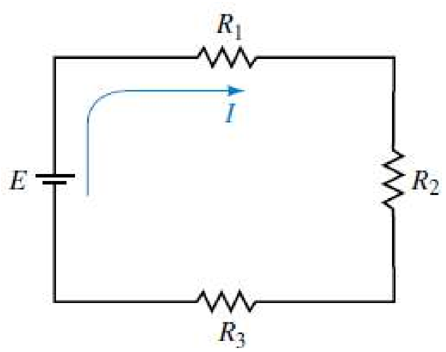


Figure 2.1: Series circuit.

The current is the same everywhere in a series circuit.

2.1.1 Resistors in series

The equivalent resistance of any number of resistors connected in series is the sum of the individual resistance.

For N resistors in series then

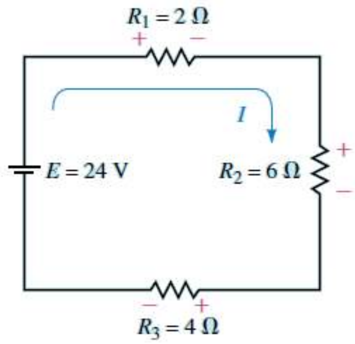
$$R_{eq} = R_1 + R_2 + ... + R_N = \sum_{n=1}^{N} R_n$$

Example 2.1:

For the circuit shown below, find the following quantities:

1. Total resistance (R_T)
2. Circuit current (I)
3. Voltage across each resistor
4. Power dissipated by each resistor

5. Power delivered to the circuit by the voltage source.



Solution:

1) $R_T = R_1 + R_2 + R_3 = 2 + 6 + 4 = 12\Omega$

2) $I = \dfrac{E}{R_T} = \dfrac{24}{12} = 2A$

3) $v_1 = IR_1 = 2*2 = 4v$

$v_2 = IR_2 = 2*6 = 12v$

$v_3 = IR_3 = 2*4 = 8v$

4)

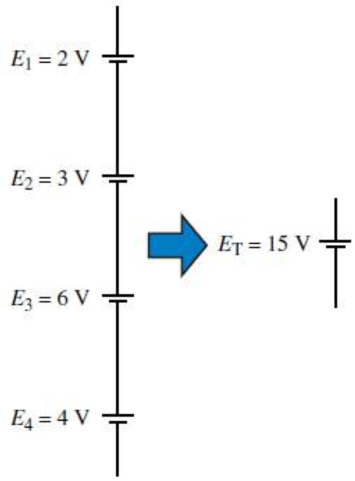
$$P_1 = I^2 R_1 = (2)^2 2 = 8w$$
$$P_2 = I^2 R_2 = (2)^2 6 = 24w$$
$$P_3 = I^2 R_3 = (2)^2 4 = 16w$$

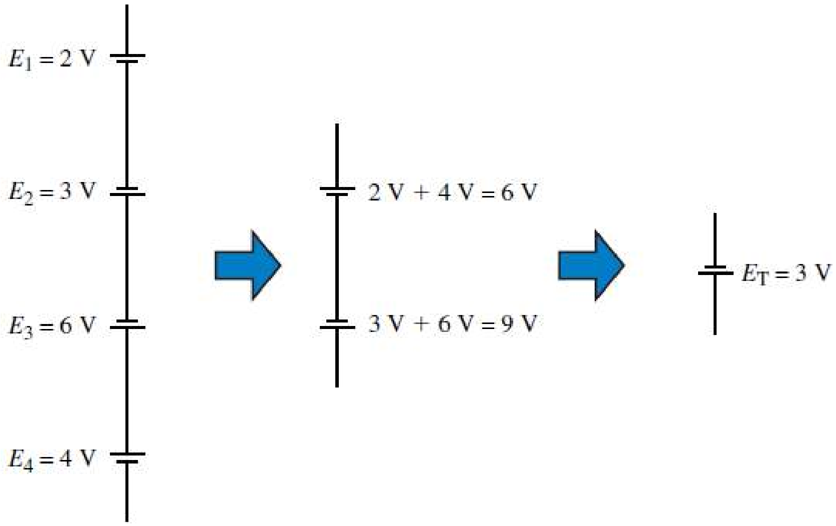
5) $P = V * I = 24*2 = 48w$

2.1.2 Voltage sources in series

If a circuit has more than one voltage source in series, then the voltage sources may effectively be replaced by a single source having a value that is the sum or difference of the individual sources.

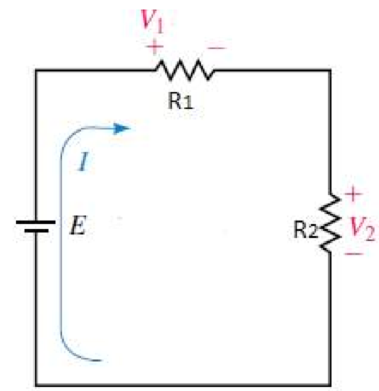
Example 2.2:





2.1.3 The Voltage Divider Rule

Consider the series circuit shown below:



We have:

$$R_T = R_1 + R_2$$

By applying ohm's law

$$v_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1$$

Similarly

$$v_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2$$

In general, the voltage drop across any resistance in the series circuit can be calculated as

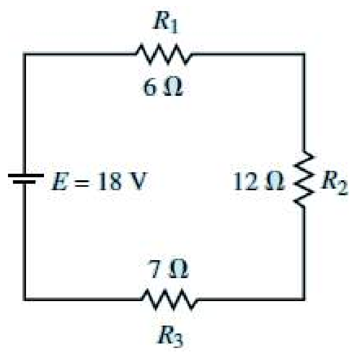
$$v_x = IR_x$$

Hence, we can write:

$$\boxed{v_x = \left(\frac{E}{R_T}\right)R_x} \rightarrow \text{Voltage divider rule}$$

Example 2.3:

Use voltage divider rule to determine the voltage across each of the resistors in the circuit shown below. Show that the summation of voltage drops is equal to the applied voltage rise in the circuit.



Solution:

$$R_T = R_1 + R_2 + R_3 = 6 + 12 + 7 = 25\Omega$$

$$v_1 = \frac{R_1}{R_T} E = \left(\frac{6}{25}\right) 18 = 4.32v$$

$$v_2 = \frac{R_2}{R_T} E = \left(\frac{12}{25}\right) 18 = 8.64v$$

$$v_3 = \frac{R_3}{R_T} E = \left(\frac{7}{25}\right) 18 = 5.04v$$

The total voltage drop is the summation

$$V_T = V_1 + V_2 + V_3 = 4.32 + 8.64 + 5.04 = 18v = E$$

2.2 Circuit ground

Ground is simply an "arbitrary electrical point of reference" or "common point" in a circuit. It is used for safety purposes. The standard symbol for circuit ground is shown in Figure 2.2.

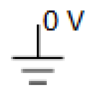


Figure 2.2: Circuit ground

Using the ground symbol in this manner usually allows the circuit to be sketched more simply

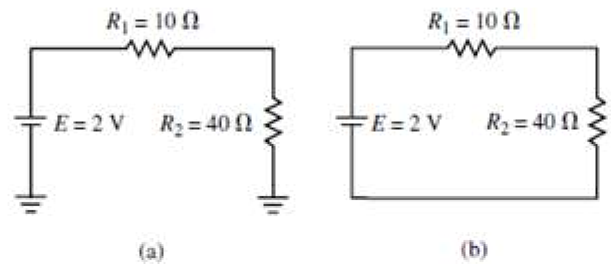
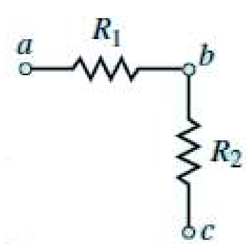


Figure 2.3: Equivalent circuits

Example 2.4:

For the circuit shown below, determine the voltages V_{ab} , and V_{cb} given that $V_a = +5$ V, $V_b = +3$ V, and $V_c = -8$ V.

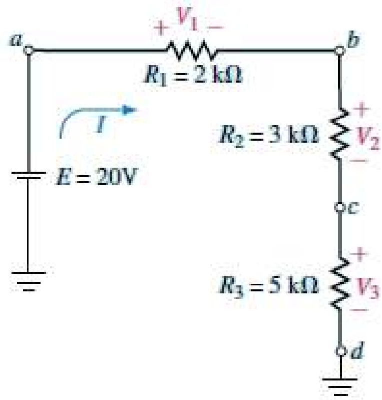


Solution:

$V_{ab} = V_a - V_b = +5 - (+3) = +2v$
$V_{cb} = V_c - V_b = -8 - (+3) = -11v$

Example 2.5:

For the circuit shown below, determine the voltages v_a , v_b , v_c and v_d .



Solution:

$$v_1 = E * \frac{R_1}{R_1 + R_2 + R_3} = 20 * \frac{2k\Omega}{10k\Omega} = 4v$$

$$v_2 = E * \frac{R_2}{R_1 + R_2 + R_3} = 20 * \frac{3k\Omega}{10k\Omega} = 6v$$

$$v_3 = E * \frac{R_3}{R_1 + R_2 + R_3} = 20 * \frac{5k\Omega}{10k\Omega} = 10v$$

By applying KVL

$$v_a = E = 20v$$

$$v_b = v_2 + v_3 = 6 + 10 = 16v$$

$$v_c = 10v$$

$$v_d = 0v$$

2.3 Parallel circuits

Elements or branches are said to be in a parallel connection when they have exactly two nodes in common, as shown in the figure 2.4.

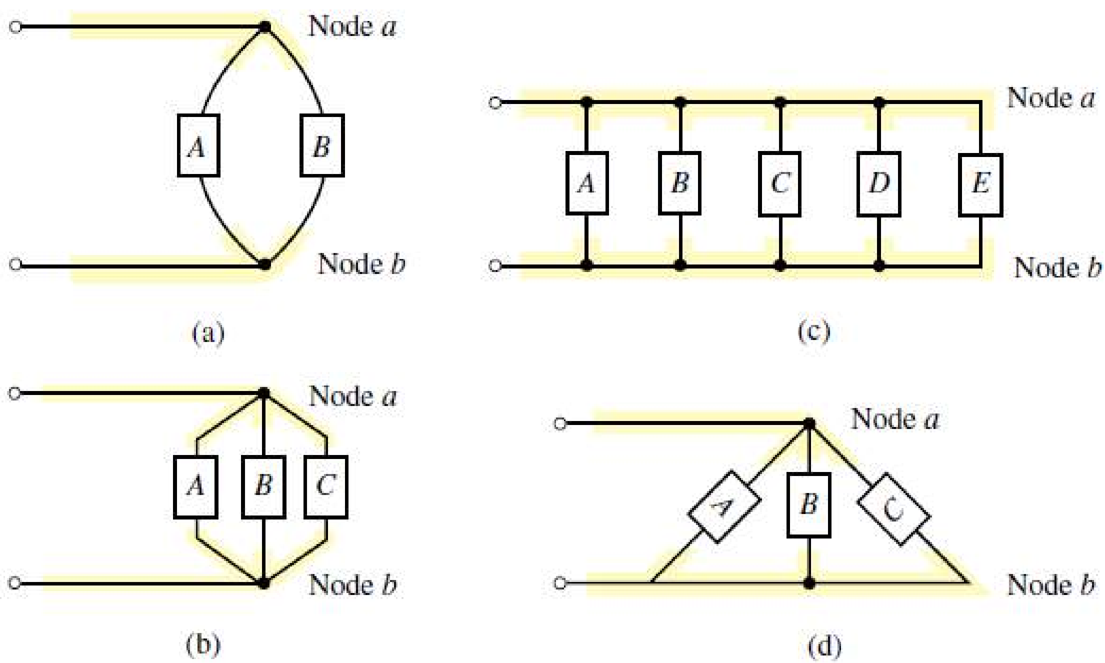


Figure 2.4: Parallel elements

2.3.1 Resistors in parallel

For N resistors connected in parallel as shown in figure 2.5, the equivalent resistance is

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}$$

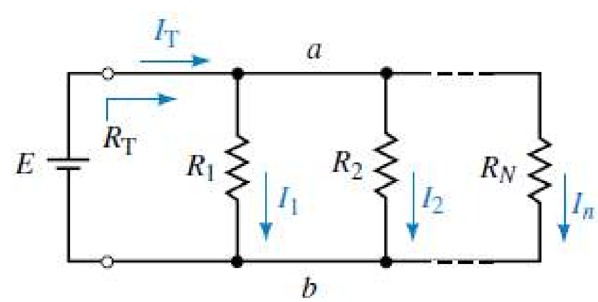


Figure 2. 5: Parallel resistors

Note: The total resistance (R_T) of parallel resistance is always less than the value of the smallest resistor.

Note: For parallel networks, it is common to use the idea of conductance in the circuit analysis rather than resistance. The conductance (G) is defined as:

$$G = \frac{1}{R}$$

So, we can write the total conductance (G_T) for the parallel circuit shown as:

$$G_T = G_1 + G_2 + G_3$$

$$\Rightarrow \qquad R_T = \frac{1}{G_T}$$

Special cases

1. For two resistors in parallel, R_T is given by

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

2. For three resistors in parallel, R_T is given by:

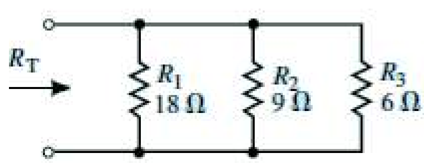
$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

3. For equal resistors in parallel (i.e. $R_1 = R_2 = R_3 .... = R_N = R$), then R_T is given by:

$$R_T = \frac{R}{N}$$

Example 2.6:

Determine the total resistance for the circuit shown below.



Solution

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{18} + \frac{1}{9} + \frac{1}{6}$$

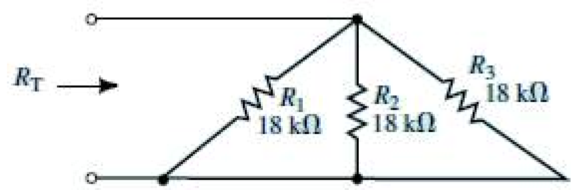
$$\frac{1}{R_T} = 0.33 \Rightarrow R_T = \frac{1}{0.33} = 3\Omega$$

OR

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} = \frac{18*9*6}{162+108+54} = \frac{972}{324} = 3\Omega$$

Example 2.7:

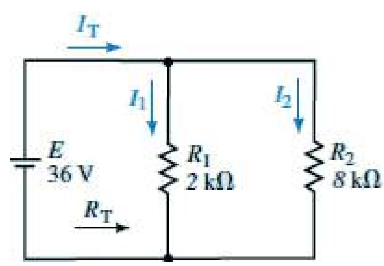
For the circuit shown below, calculate the total resistance



Solution:

$$R_T = \frac{R}{N} = \frac{18k\,\Omega}{3} = 6k\,\Omega$$

Example 2.8:



For the circuit shown above, determine the following quantities

a) The total resistance (R_T)

b) The total current (I_T)

c) I_1 and I_2

d) Power delivered by the voltage source

e) Power dissipated by the resistors

Solution:

a) $R_T = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{2k\Omega * 8k\Omega}{10k\Omega} = 1.6k\Omega$

b) $I_T = \dfrac{E}{R_T} = \dfrac{36v}{1.6*10^3\Omega} = 22.5mA$

c) Since the resistors in parallel with the voltage source, then

$E = V_{R1} = V_{R2} = 36v$

$I_1 = \dfrac{V_{R1}}{R_1} = \dfrac{36v}{2*10^3\Omega} = 18mA$

$I_2 = \dfrac{V_{R2}}{R_2} = \dfrac{36v}{8*10^3\Omega} = 4.5mA$

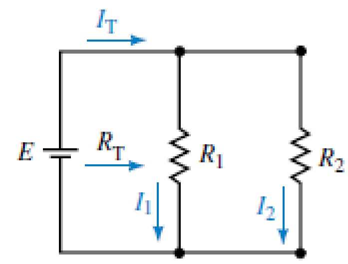
d) $P_T = E * I_T = 36*22.5*10^{-3} = 810mw$

e) $P_1 = I_1^2 * R_1 = (18*10^{-3})^2 * 2*10^3 = 648mw$

$P_2 = I_2^2 * R_2 = (4.5*10^{-3})^2 * 8*10^3 = 162mw$

2.3.2 Current divider rule

Consider the circuit shown below,



$$I_T = \frac{E}{R_T} \quad \text{and} \quad R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{E}{R_1} = \frac{I_T R_T}{R_1} = \frac{I_T \dfrac{R_1 R_2}{R_1 + R_2}}{R_1}$$

$$\boxed{\therefore I_1 = I_T \frac{R_2}{R_1 + R_2}}$$

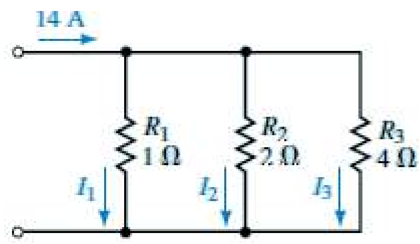
Similarly, for I_2

$$I_2 = \frac{E}{R_2} = \frac{I_T R_T}{R_2} = \frac{I_T \dfrac{R_1 R_2}{R_1 + R_2}}{R_2}$$

$$\boxed{\therefore I_2 = I_T \frac{R_1}{R_1 + R_2}}$$

Example 2.9

For the circuit shown below, find I_1 , I_2 and I_3



Solution:

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$R_T = \frac{8}{2+4+8} = \frac{8}{14} = 0.571\Omega$$

$$I_1 = \frac{I_T R_T}{R_1} = \frac{14*0.571}{1} = 8A$$

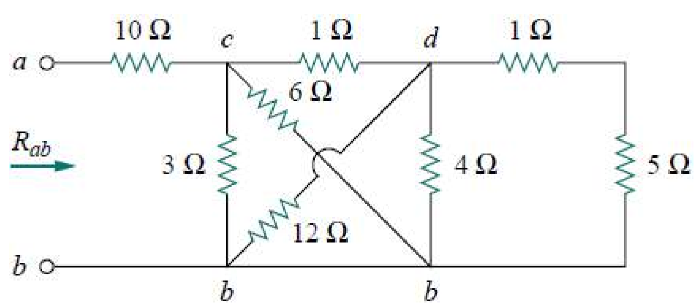
$$I_2 = \frac{I_T R_T}{R_2} = \frac{14*0.571}{2} = 4A$$

$$I_3 = \frac{I_T R_T}{R_3} = \frac{14*0.571}{4} = 2A$$

2.4 Series-Parallel circuits

Example 2.10:

For the circuit shown below, calculate the equivalent resistance R_{ab}

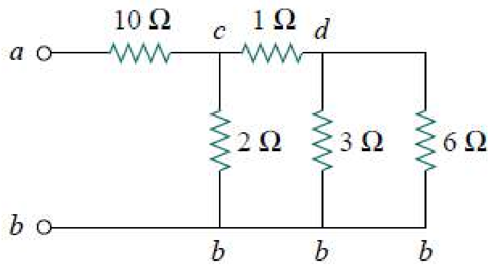


Solution

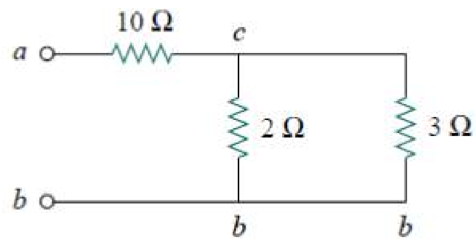
$$1\Omega + 5\Omega = 6\Omega$$

$$12\Omega // 4\Omega = \frac{12*4}{16} = 3\Omega$$

$$3\Omega // 6\Omega = \frac{3*6}{9} = 2\Omega$$

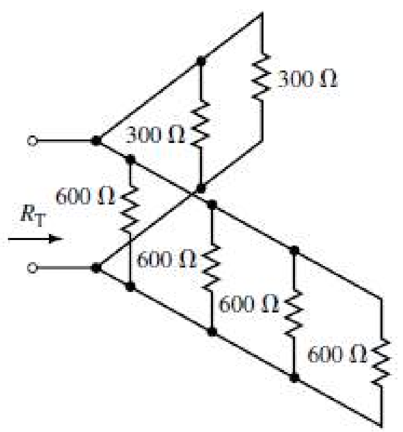


$$3\Omega // 6\Omega + 1\Omega = \frac{3*6}{9} + 1 = 2\Omega + 1\Omega = 3\Omega$$



$$\therefore R_{ab} = 3\Omega // 2\Omega + 10\Omega = \frac{3*2}{5} + 1 = 1.2\Omega + 10\Omega = 11.2\Omega$$

H.w: Find R_T for the circuit shown below

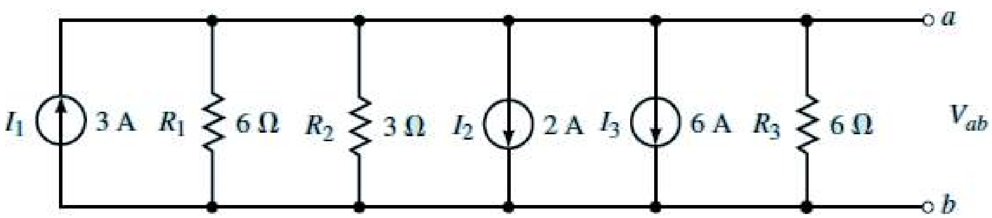


Answer: $R_T = 75\Omega$

2.5 *Current sources in parallel*

If two or more current sources are in parallel, they may all be replaced by one current source having the magnitude and direction of the resultant, which can be found by summing the currents in one direction and subtracting the sum of the currents in the opposite direction.

Example 2.11:
Simplify the circuit shown below

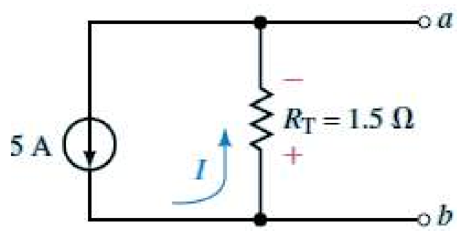


Solution
Since all of the current sources are in parallel, they can be replaced by a single current source.

$I = 3 - 2 - 6 = -5A$

$R_T = 6\Omega // 3\Omega // 6\Omega = 1.5\Omega$

The equivalent circuit is shown below



2.5 Source conversions

A voltage source with voltage E and series resistor R_s can be replaced by a current source with a current I and parallel resistor R_s as shown in figure 2.6.

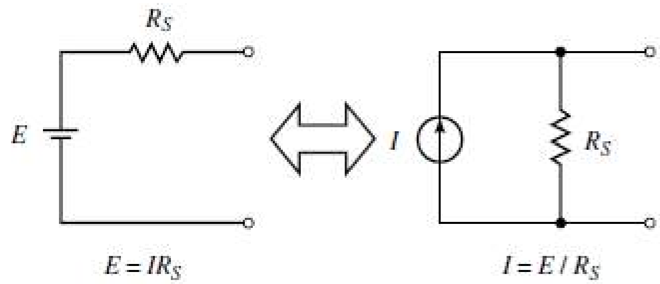
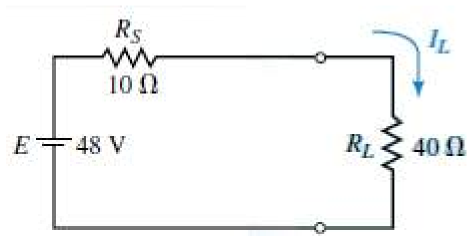


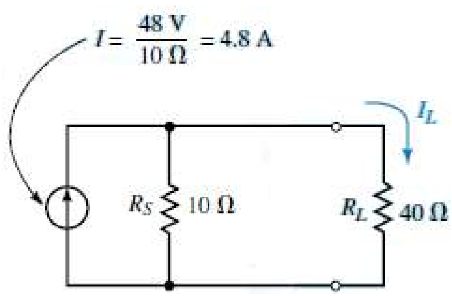
Figure 2.6: Source conversions

Example 2.12:

Convert the voltage source of the figure below into a current source

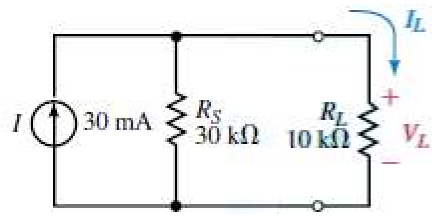


Solution:

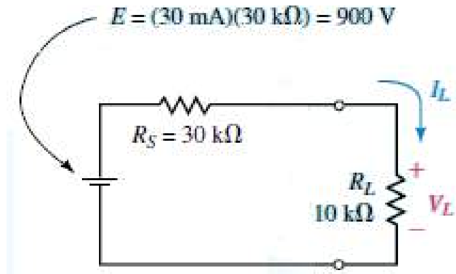


Example 2.13:

Convert the current source of the figure below into a voltage source

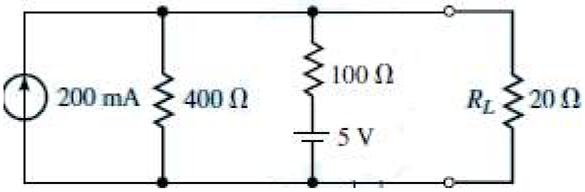


Solution:

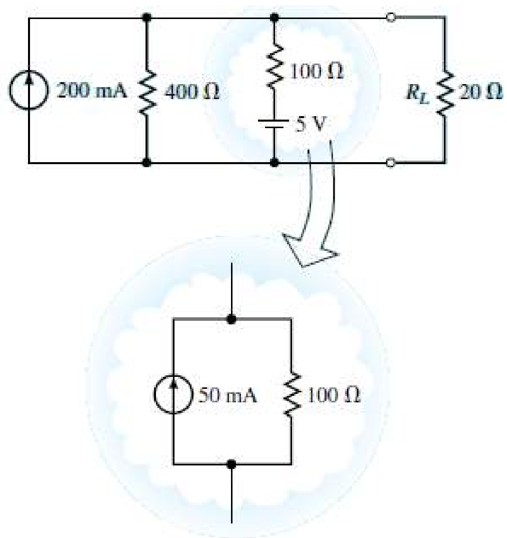


Example 2.14

Find the current through the resistor R_L

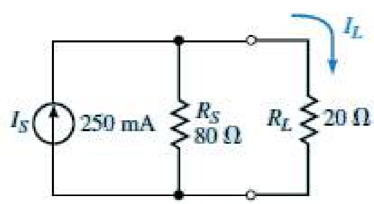


Solution:



$$I_S = 200mA + 50mA = 250mA$$

The simplified circuit is shown below



$$I_L = \frac{I_s * R_s}{R_L + R_s} = \frac{250*10^{-3}*80}{100} = 200mA$$

2.6 Wye-Delta Transformations

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in figure 2.7.

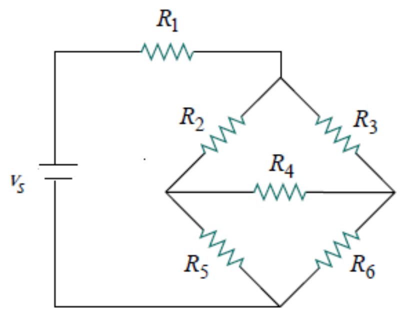


Figure 2.7: Bridge circuit

In this circuit, $R_1, R_2 ....R_6$ are neither in parallel nor in series. Many circuits of the type shown in Fig. 2.7 can be simplified by using three-terminal equivalent networks. These are the wye (Y) network shown in Fig. 2.8 and the delta (Δ) network shown in Fig. 2.9.

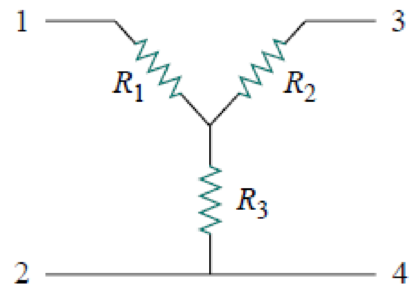


Figure 2.8: The form of wye (Y) network

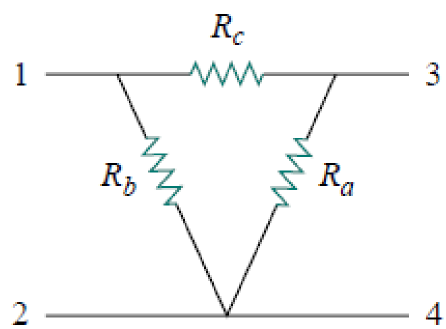


Figure 2.9: The form of delta (Δ) network

Delta to Wye conversion

We have delta (Δ) connected circuit and want to get its equivalent wye (Y) circuit

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

In general: ***Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.***

Wye to Delta conversion

We have wye (Y) connected circuit and want to get its equivalent delta (Δ) circuit

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

In general: ***Each resistor in the Δ network is the sum of all possible of Y resistors taken two at a time, divided by the opposite Y resistor.***

The Y and Δ networks are said to be balanced when

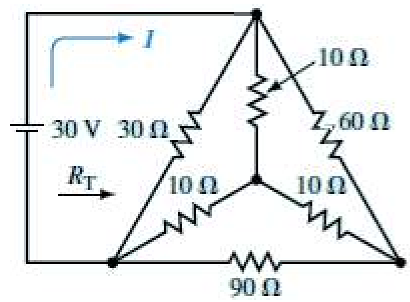
$R_1 = R_2 = R_3 = R_Y$, $\quad\quad R_a = R_b = R_c = R_\Delta$

Under these conditions, conversion formulas become:

$$\boxed{R_Y = \frac{R_\Delta}{3}} \quad \text{OR} \quad \boxed{R_\Delta = 3R_Y}$$

Example 2.15:

For the circuit shown below, calculate the total resistance (R_T) and the total current I

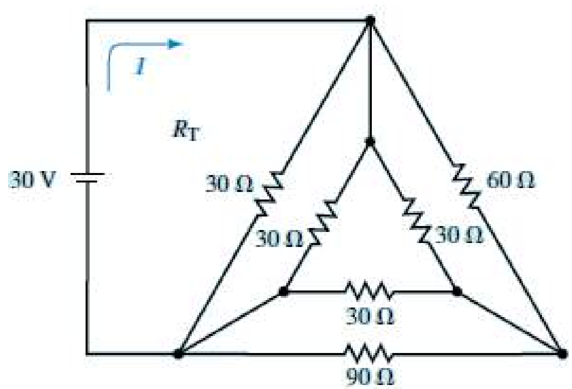


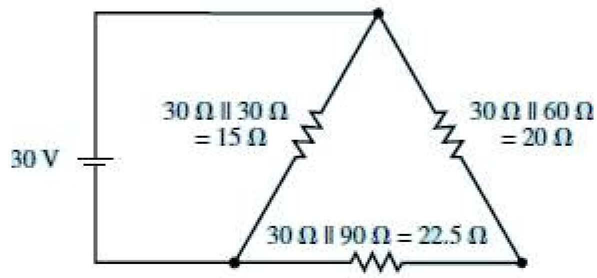
Solution:

We convert the" Δ " into its equivalent "Y "

$R_\Delta = 3R_Y = 3*10 = 30\Omega$

The resulting circuit is shown below



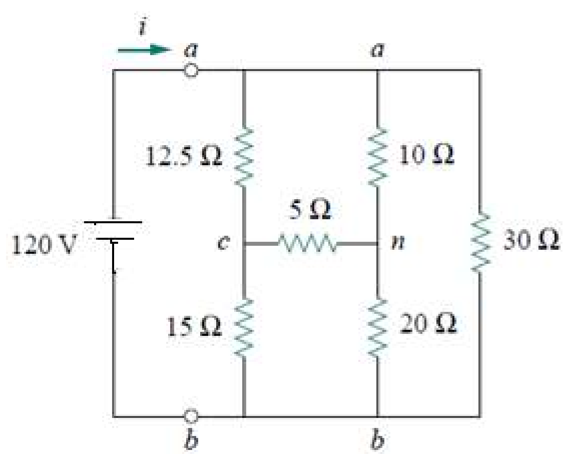


$R_T = (20 + 20.5) // 15 = 11.09\Omega$

$I = \frac{E}{R_T} = \frac{30v}{11.09\Omega} = 2.706A$

Example 2.16:

For the circuit shown below, calculate the equivalent resistance R_{ab} and the current i



Solution:

We convert the Y network $(5\Omega, 10\Omega, 20\Omega)$ into its equivalent "Δ"

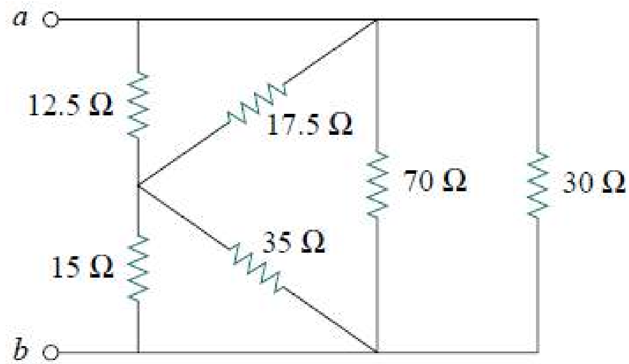
$R_1 = 10\Omega$, $R_2 = 20\Omega$, $R_3 = 5\Omega$

$$R_a = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1} = \frac{10*20 + 10*5 + 20*5}{10} = \frac{350}{10} = 35\Omega$$

$$R_b = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2} = \frac{350}{20} = 17.5\Omega$$

$$R_c = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3} = \frac{350}{5} = 70\Omega$$

The resulting circuit is shown below



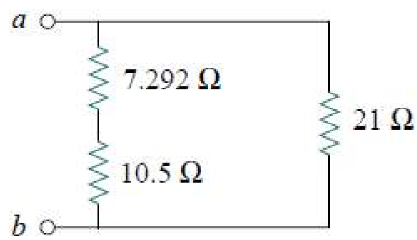
Now we can see that

$17.5\Omega // 12.5\Omega = 7.292\Omega$

$35 // 15 = 10.5\Omega$

$70 // 30 = 21\Omega$

The resulting circuit is shown below



From the final circuit

$$R_{ab} = (7.292 + 10.5 // 21) = \frac{17.792 * 21}{17.792 + 21} = 9.632\Omega$$

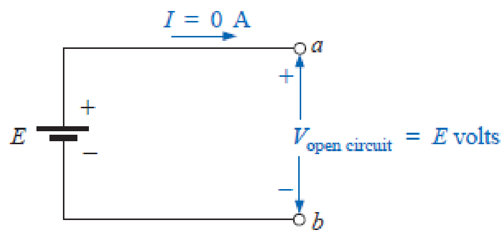
$$i = \frac{E}{R_{ab}} = \frac{120v}{9.632\Omega} = 12.458A$$

2.7 Open and short circuits

We frequently need to use the open and short circuits in the analysis of electric networks.

- **Open circuit**
  An open circuit is simply two isolated terminals not connected by an element of any kind.

  See the circuit shown below, with open circuit terminals a and b.



$$V_{\substack{open\\circuit}} = V_{OC} = V_{ab} = E$$

In general:
An open circuit can have a potential difference (voltage) across its terminals, but the current is always zero ampere.

- **Short circuit**
  A short circuit is a direct connection of zero ohms across an element or combination of elements.

  Consider the circuit shown, with a short circuit across the resistor R_2

$$I_{Sc} = I_T = \frac{E}{R_1} \text{ since } I_{R_2} = 0$$

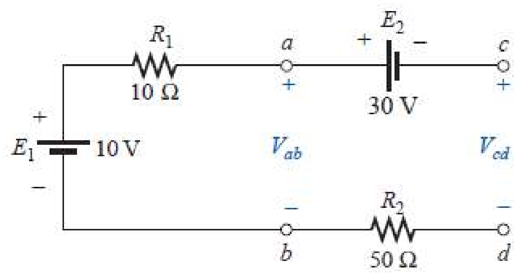
$$V_{\substack{short\\circuit}} = V_{SC} = 0$$

In general:

A short circuit can carry a current of any level but the potential difference (voltage) across its terminals is always zero volts.

Example 2.17:

Determine the voltages V_{ab} and V_{cd} for the circuit shown below



Solution:

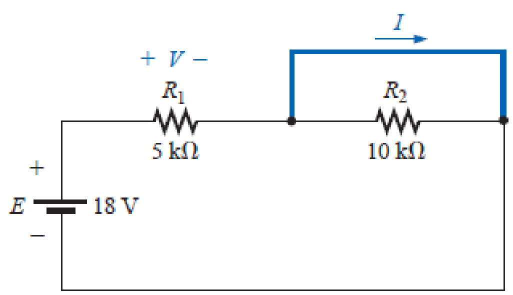
Since the circuit is open, the current is zero.

$$V_{ab} = E_1 = 10v$$

$$V_{ab} - E_2 - V_{cd} = 0 \quad \rightarrow V_{cd} = 10 - 30 = -20v$$

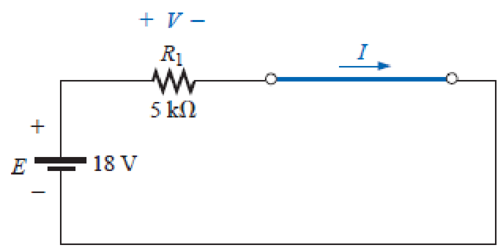
Example 2.18:

Calculate the current I and the voltage V for the circuit shown below



Solution:

No current through R_2, thus the resulting circuit will be as shown below



$$I = \frac{E}{R_1} = \frac{18v}{5*10^3} = 3.6mA$$

$$V = IR_1 = 3.6*10^{-3}*5*10^3 = 18v$$