$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = \ldots + x(-1)h(1) + x(0)h(0) + x(1)h(-1) + x(2)h(-2) + \ldots$$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = \ldots + x(-1)h(2) + x(0)h(1) + x(1)h(0) + x(2)h(-1) + \ldots$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = \ldots + x(-1)h(3) + x(0)h(2) + x(1)h(1) + x(2)h(0) + \ldots$$

$\ldots$

Digital convolution using the graphical method.

Step 1. Obtain the reversed sequence $h(-k)$.

Step 2. Shift $h(-k)$ by $|n|$ samples to get $h(n-k)$. If $n \geq 0$, $h(-k)$ will be shifted to the right by n samples; but if $n < 0$, $h(-k)$ will be shifted to the left by $|n|$ samples.

Step 3. Perform the convolution sum that is the sum of the products of two sequences $x(k)$ and $h(n-k)$ to get $y(n)$.

Step 4. Repeat steps 1 to 3 for the next convolution value $y(n)$.

Example

Given a sequence,

$$h(k) = \begin{cases} 3, & k = 0,1 \\ 1, & k = 2,3 \\ 0 & elsewhere \end{cases}$$

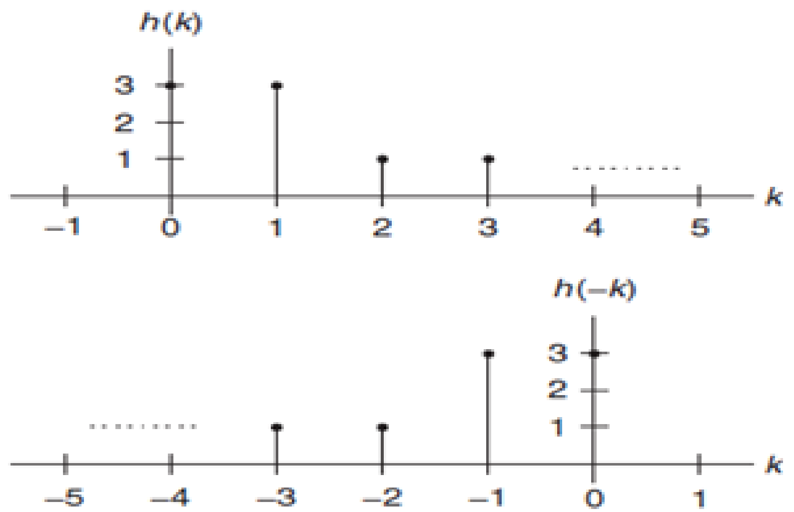
where k is the time index or sample number,

a) Sketch the sequence *h(k)* and reversed sequence h(- k).

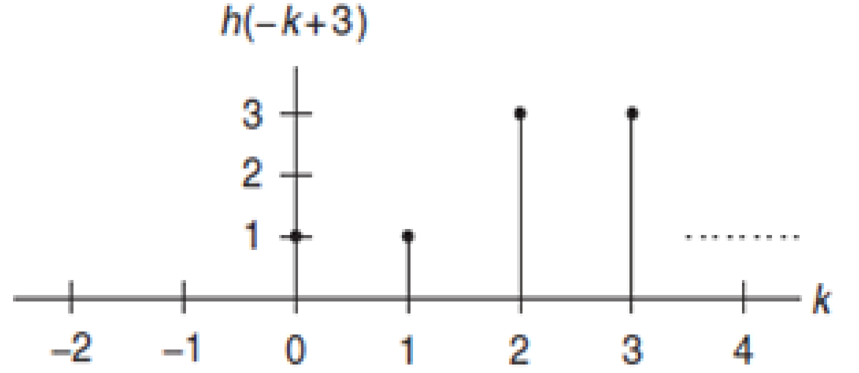
b) Sketch the shifted sequences *h(-k +3)* and *h(- k - 2)*.

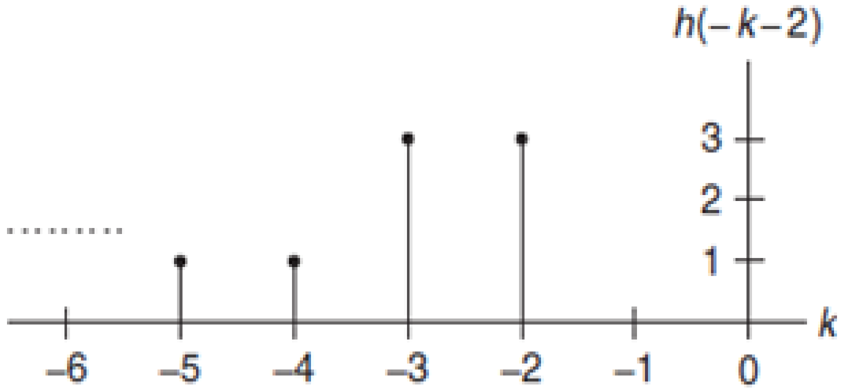
Solution:

a)



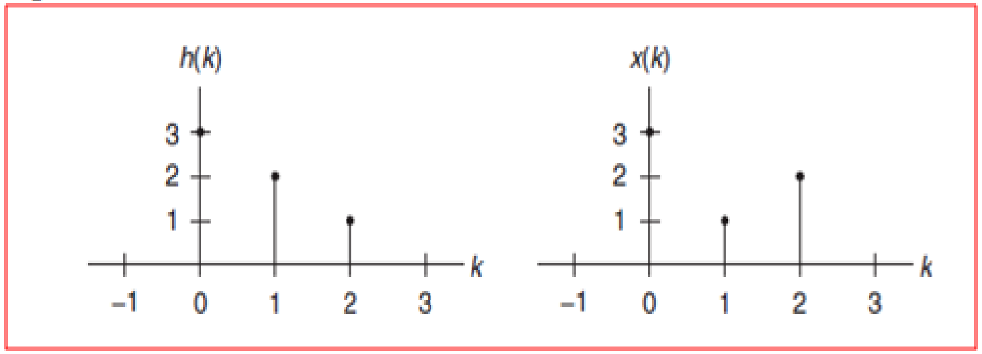
b)





Example

Using the following sequences defined in the Figure below, evaluate the digital convolution



a) By the graphical method.

b) By applying the formula directly.

sum of product of $x(k)$ and $h(-k)$: $y(0) = 3 \times 3 = 9$

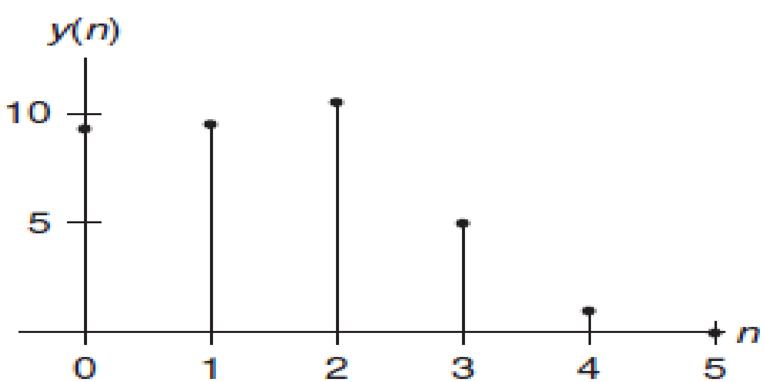
sum of product of $x(k)$ and $h(1-k)$: $y(1) = 1 \times 3 + 3 \times 2 = 9$

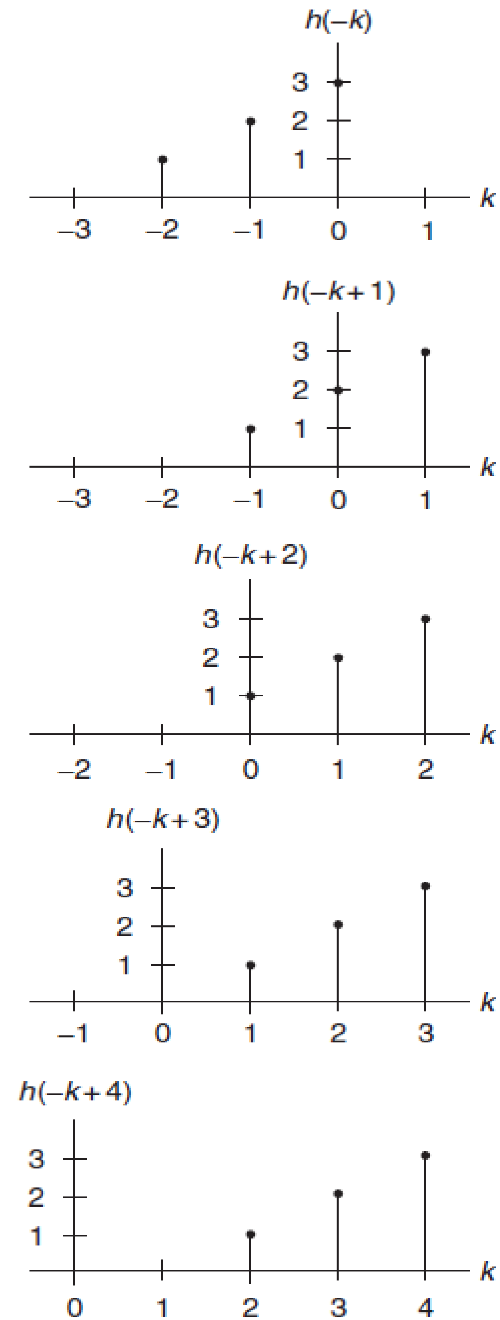
sum of product of $x(k)$ and $h(2-k)$: $y(2) = 2 \times 3 + 1 \times 2 + 3 \times 1 = 11$

sum of product of $x(k)$ and $h(3-k)$: $y(3) = 2 \times 2 + 1 \times 1 = 5$

sum of product of $x(k)$ and $h(4-k)$: $y(4) = 2 \times 1 = 2$

sum of product of $x(k)$ and $h(5-k)$: $y(n) = 0$ for $n > 4$, since sequences $x(k)$ and $h(n-k)$ do not overlap.





b) Applying Equation (4) with zero initial conditions leads to

$$y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2)$$

$$n = 0,\ y(0) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) = 3\times 3 + 1\times 0 + 2\times 0 = 9,$$

$$n = 1,\ y(1) = x(0)h(1) + x(1)h(0) + x(2)h(-1) = 3\times 2 + 1\times 3 + 2\times 0 = 9,$$

$$n = 2,\ y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0) = 3\times 1 + 1\times 2 + 2\times 3 = 11,$$

$$n = 3,\ y(3) = x(0)h(3) + x(1)h(2) + x(2)h(1) = 3\times 0 + 1\times 1 + 2\times 2 = 5.$$

$$n = 4,\ y(4) = x(0)h(4) + x(1)h(3) + x(2)h(2) = 3\times 0 + 1\times 0 + 2\times 1 = 2,$$

$$n \geq 5,\ y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) = 3\times 0 + 1\times 0 + 2\times 0 = 0.$$

Convolution using Table Method

Solution:

Digital convolution steps via the table.

Step 1. List the index k covering a sufficient range.

Step 2. List the input $x(k)$.

Step 3. Obtain the reversed sequence $h(-k)$, and align the rightmost element of $h(n-k)$ to the leftmost element of $x(k)$.

Step 4. Cross-multiply and sum the nonzero overlap terms to produce $y(n)$.

Step 5. Slide $h(n-k)$ to the right by one position.

Step 6. Repeat step 4; stop if all the output values are zero or if required.

Convolution sum using the table method.

k:	−2	−1	0	1	2	3	4	5	
$x(k)$:			3	1	2				
$h(-k)$:	1	2	3						$y(0) = 3 \times 3 = 9$
$h(1-k)$		1	2	3					$y(1) = 3 \times 2 + 1 \times 3 = 9$
$h(2-k)$			1	2	3				$y(2) = 3 \times 1 + 1 \times 2 + 2 \times 3 = 11$
$h(3-k)$				1	2	3			$y(3) = 1 \times 1 + 2 \times 2 = 5$
$h(4-k)$					1	2	3		$y(4) = 2 \times 1 = 2$
$h(5-k)$						1	2	3	$y(5) = 0$ (no overlap)