Classification of Discrete – Time Systems:-

•Static versus dynamic systems.

Static systems	Dynamic systems
$y(n) = ax(n)$ $y(n) = nx(n) + bx^3(n)$	$y(n) = x(n) + 3x(n-1)$ $y(n) = \sum_{k=0}^{n} x(n-k)$

•Time – Invariant versus Time – Variant systems

A relaxed system T is time invariant or shift invariant if and only if

$$x(n) \xrightarrow{T} y(n)$$

Implies that

$$x(n-k) \xrightarrow{T} y(n-k)$$

For every input signal $x(n)$ and every time shift k.

In general, we can write the output as

$$y(n,k) = T[x(n-k)]$$

Now if this output $y(n,k) = y(n-k)$, for all possible values of k, the system is time invariant. On the other hand, if the output $y(n,k) \neq y(n-k)$, even for one value of k, the system is time variant.

Example:-

The system described by the input – output equation

$$y(n) = T[x(n)] = x(n) - x(n-1)$$

Now if the input is delayed by k units in time and applied to the system

$$y(n,k) = x(n-k) - x(n-k-1)$$

On the other hand, if we delay $y(n)$ by k units in time, we obtain

$$y(n-k) = x(n-k) - x(n-k-1)$$

Therefore, $y(n,k) = y(n-k)$ and the system is time invariant.

Example:-

The system described by the input – output equation

$$y(n) = T[x(n)] = x(-n)$$

The response of this system to $x(n-k)$ is

Only delay the input by -k

$$y(n,k) = T[x(n-k)] = x(-n-k)$$

Now, if we delay the output $y(n)$ by k units in time, the result will be

$$y(n-k) = x(-n+k)$$

Since $y(n,k) \neq y(n-k)$, the system is time variant.

c) Linear versus nonlinear systems.

A relaxed T system is linear if and only if

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

For any arbitrary input sequences $x_1(n)$ and $x_2(n)$, and any arbitrary constants a_1 and a_2.

Example:-

Determine if the systems described by the following input – output equations are linear or nonlinear.

a) $y(n) = nx(n)$

b) $y(n) = x^2(n)$

Solution:-

a) For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$y_1(n) = nx_1(n)$$
$$y_2(n) = nx_2(n)$$

A linear combination of the two input sequences results in the output

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)]$$

A linear combination of the $y_1(n)$ and $y_2(n)$ results in

$$a_1y_1(n) + a_2y_2(n) = n[a_1x_1(n) + a_2x_2(n)]$$

Since $y_3(n) \equiv a_1y_1(n) + a_2y_2(n)$ the system is linear.

b) The responses of the system to two separate input signals are

$$y_1(n) = {x_1}^2(n)$$
$$y_2(n) = {x_2}^2(n)$$

The response of the system to a linear combination of these two input signals is

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= [a_1 x_1(n) + a_2 x_2(n)]^2$$
$$= a_1{}^2 x_1{}^2(n) + 2a_1 a_2 x_1(n) x_2(n) + a_2{}^2 x_2{}^2(n)$$

On the other hand, if the system is linear, it would produce a linear combination of the two outputs

$$a_1 y_1(n) + a_2 y_2(n) = a_1 x_1{}^2(n) + a_2 x_2{}^2(n)$$

c) Causal versus noncausal systems

In mathematical terms, the output of a causal system satisfies an equation of the form

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots]$$

Where F[.] is some arbitrary function.

If a system does not satisfy this definition, it is called noncausal. Such a system has an output that depends not only on present and past inputs but also on future inputs.

Example:-

Determine if the systems described by the following input – output equations are causal or noncausal.

a) $y(n) = x(n) - x(n-1)$

b) $y(n) = \sum_{k=-\infty}^{n} x(k)$

c) $y(n) = ax(n)$

d) $y(n) = x(n) + 3x(n+4)$

d) $y(n) = x(n^2)$

f) $y(n) = x(2n)$

g) $y(n) = x(-n)$

Solution:-

The systems described by parts (a), (b), and (c) are causal.
The systems described by rest parts are noncausal.

d) Stable versus unstable systems

An arbitrary relaxed system is said to be bounded input – bounded output (BIBO) stable if and only if every bounded input produces a bounded output.

The conditions that the input sequence $x(n)$ and the output sequence $y(n)$ are bounded is translated mathematically to mean that there exist some finite numbers,
Say M_x and M_y such that

$$|x(n)| \le M_x < \infty \qquad\qquad |y(n)| \le M_y < \infty$$

For all n. If, for some bounded input sequence $x(n)$, the output is unbounded (infinite), the system is classified as unstable.

Example:-

Consider the nonlinear system described by the input – output equation

$$y(n) = y^2(n-1) + x(n)$$

As an input sequence we select the bounded signal

$$x(n) = C\delta(n)$$

Where C is a constant. We also assume that $y(-1) = 0$. Then the output sequence is

$$y(0) = C, \qquad y(1) = C^2, \qquad \ldots\ldots y(n) = C^{2n}$$

Clearly, the output is unbounded when $1 < |C| < \infty$. Therefore, the system is unstable.