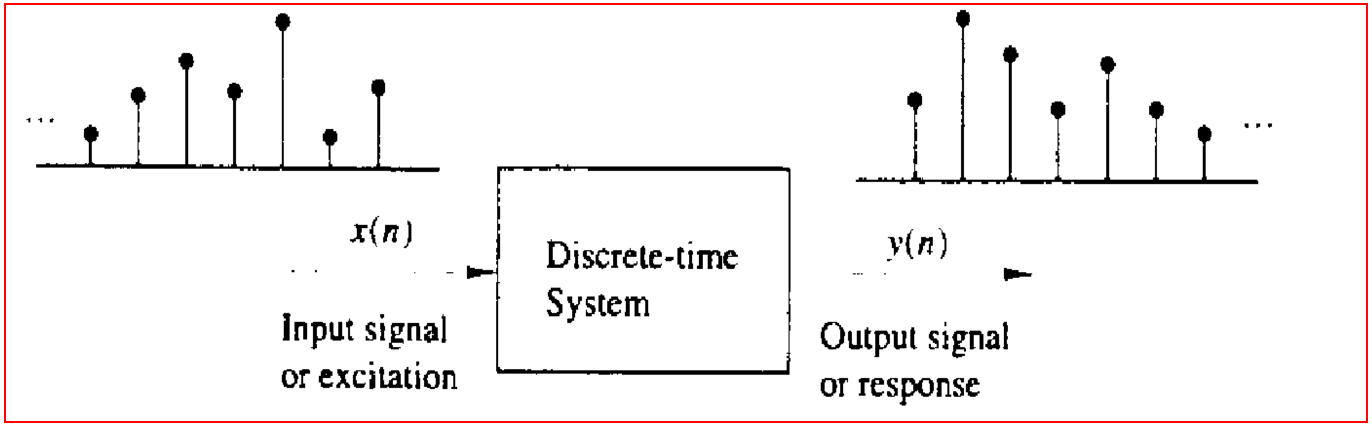
Discrete Time Systems

In many applications of digital signal processing we wish to design a device or an algorithm that performs some prescribed operation on discrete – time signal. Such a device or algorithm is called a discrete – time system. More specifically, a discrete – time system is a device or algorithm that operates on a discrete – time signal called the input or excitation, according to some well – defined rule, to produce another discrete – time signal called the output or response of the system.

Input – output Description of Systems

The input – output description of a discrete – time system consists of a mathematical expression or a rule, which explicitly defines the relation between the input and output signals.



Block diagram representation of a discrete – time system

Example

Determine the response of the following systems to the input signal

$$x(n) = \begin{cases} |n|, & -3 \le n \le 3 \\ 0, & otherwise \end{cases}$$

a) $y(n) = x(n)$

b) $y(n) = x(n-1)$

c) $y(n) = x(n+1)$

d) $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

Solution

First, we determine explicitly the sample values of the input signal

$$x(n) = \{\ldots\ldots 0,3,2,1,0,1,2,3,0\ \ldots\ldots\}$$

Next, we determine the output of each system using its input – output relationship.

- In this case the output is exactly the same as the input signal. Such a system is known as the identity system.
- This system simply delays the input by one sample

$x(n)$	3	2	1	0	1	2	3
$x(n-1)$	0	3	2	1	0	1	2

•In this case the system "advances" the input one sample into the future.

$x(n)$	3	2	1	0	1	2	3
$x(n+1)$	2	1	0	1	2	3	0

•The output of this system at any time is the mean value of the present, the immediate past, and the immediate future samples

Example

The accumulator described by

$$y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \cdots$$

Is excited by the sequence $x(n) = nu(n)$. Determine its output under the condition that:

a) It is initially relaxed [i.e., $y(-1) = 0$].

b) Initially, $y(-1) = 1$.

Solution

The output of the system is defined as

$$y(n) = \sum_{k=-\infty}^{n} x(k) = \sum_{k=-\infty}^{-1} x(k) + \sum_{k=0}^{n} x(k)$$

$$= y(-1) + \sum_{k=0}^{n} x(k)$$

But

$$\sum_{k=0}^{n} x(k) = \frac{n(n+1)}{2}$$

For a

$$y(n) = \frac{n(n+1)}{2}$$

For b

$$y(n) = 1 + \frac{n(n+1)}{2}$$

Block Diagram representation of Discrete – Time Systems

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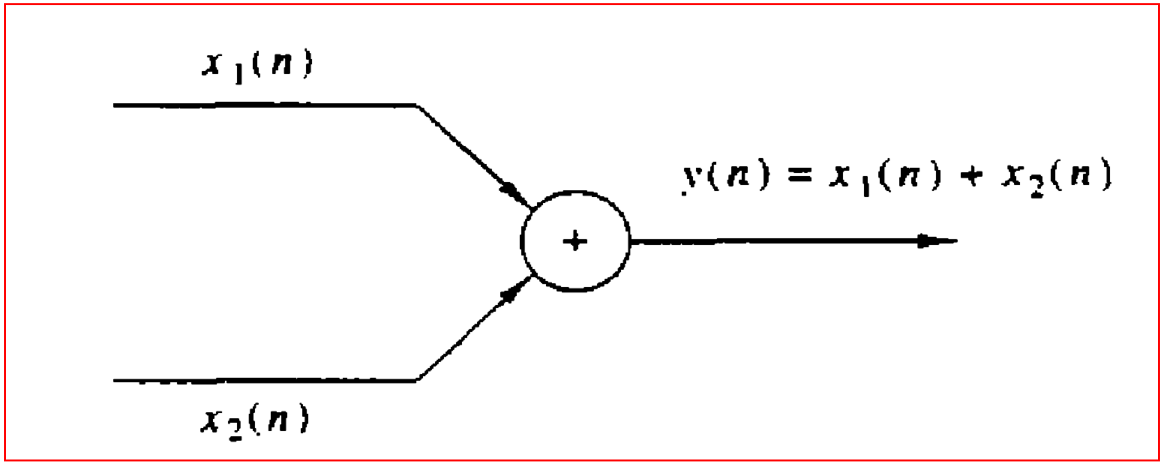
It is useful at this point to introduce a block diagram representation of discrete – time systems. For this purpose we need to define some basic building blocks that can be interconnected to form complex systems.

An adder.

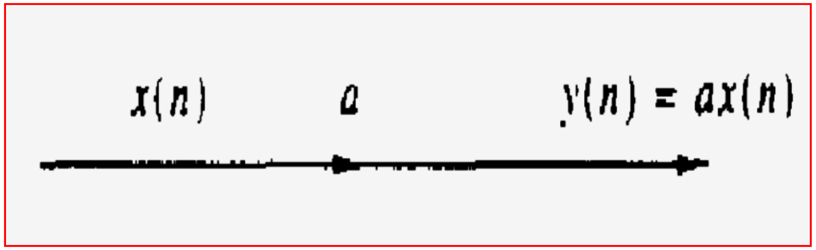
The figure below illustrate a system (adder) that performs the addition of two signal sequences to form another (the sum) sequence, which we denote as $y(n)$



Note that it is not necessary to store either one of the sequences in order to perform the addition. In other words, the addition operation is memoryless.

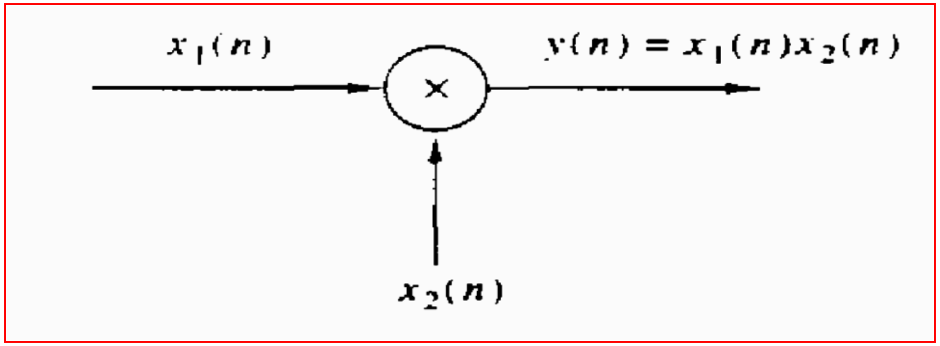
A constant multiplier.

This operation is depicted by figure below, and simply represent applying a scale factor on the input *x(n)*. Note that this operation is also memoryless.



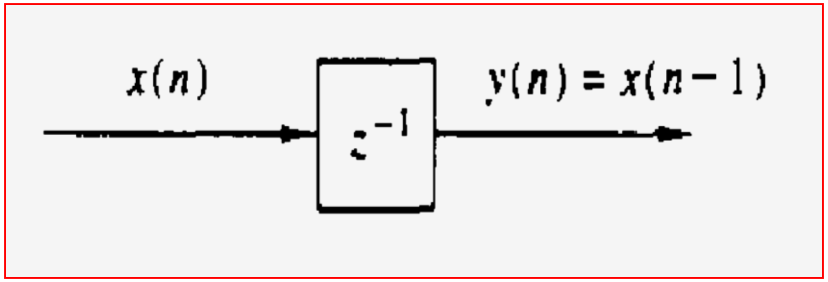
A signal multiplier.

Figure below illustrates the multiplication of two signal sequences, the multiplication operation is memoryless.



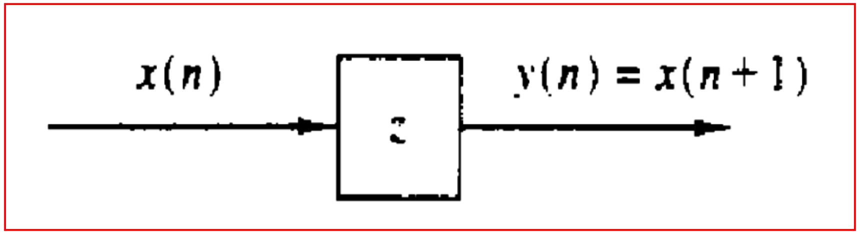
A unit delay element.

The unit delay is a special system that simply delays the signal passing through it by on sample.



A unit advance element.

In contrast to the unit delay, a unit advance moves the input a head by one sample.



Example:-

Using basic building blocks introduced above, sketch the block diagram representation of the discrete – time system described by the input – output relation.

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

Solution:-