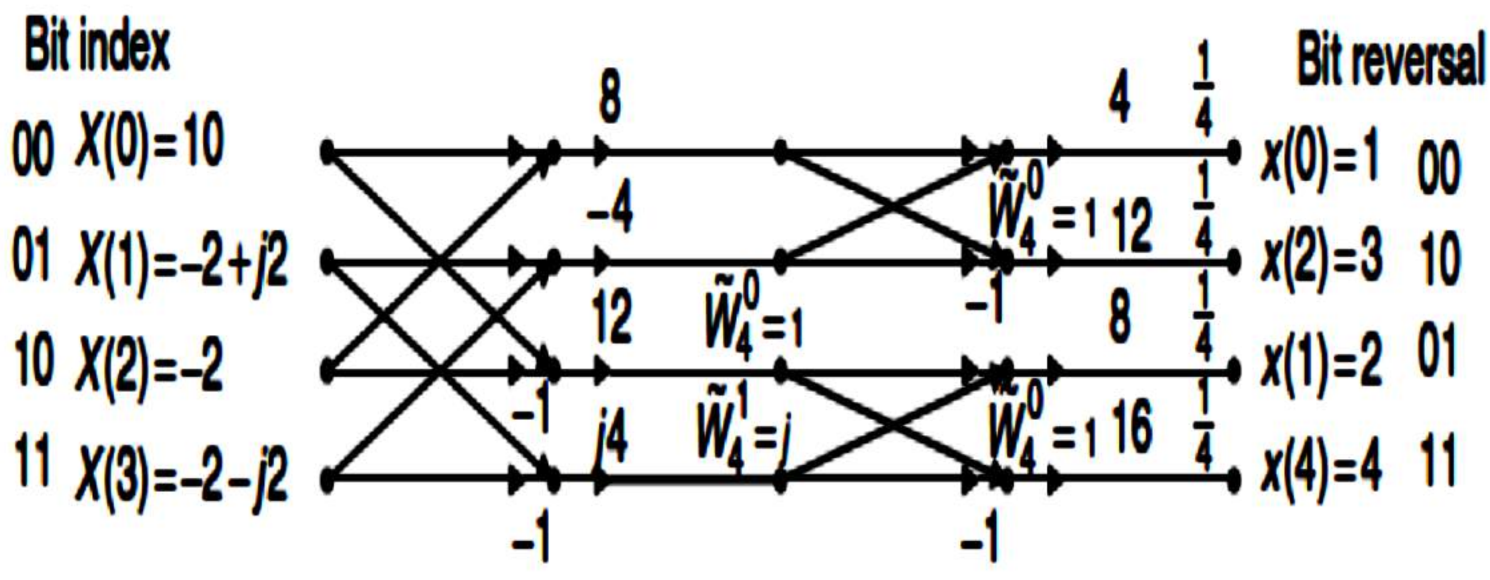
Example

Given the DFT sequence $X(k)$ for $0 \leq k \leq 3$ computed in previous example. Evaluate its inverse DFT $x(n)$ using the decimation-in-frequency FFT method.

Solution:



Method of Decimation-in-Time

In this method, we split the input sequence *x(n)* into the even indexed *x(2m)* and *x(2m + 1)*, each with N data points.

$$X(k) = \sum_{m=0}^{(N/2)-1} x(2m) W_N^{2mk} + \sum_{m=0}^{(N/2)-1} x(2m+1) W_N^k W_N^{2mk},$$

for $k = 0, 1, \ldots, N-1.$

Using the relation

$$W_N^2 = W_{N/2},$$

it follows that

$$X(k) = \sum_{m=0}^{(N/2)-1} x(2m) W_{N/2}^{mk} + W_N^k \sum_{m=0}^{(N/2)-1} x(2m+1) W_{N/2}^{mk},$$

for $k = 0, 1, \ldots, N-1.$

Define new functions as

$$G(k) = \sum_{m=0}^{(N/2)-1} x(2m) W_{N/2}^{mk} = DFT\{x(2m) \text{ with } (N/2) \text{ points}\}$$

$$H(k) = \sum_{m=0}^{(N/2)-1} x(2m+1) W_{N/2}^{mk} = DFT\{x(2m+1) \text{ with } (N/2) \text{ points}\}.$$

Note that

$$G(k) = G\left(k + \frac{N}{2}\right), \text{ for } k = 0, 1, \ldots, \frac{N}{2} - 1$$

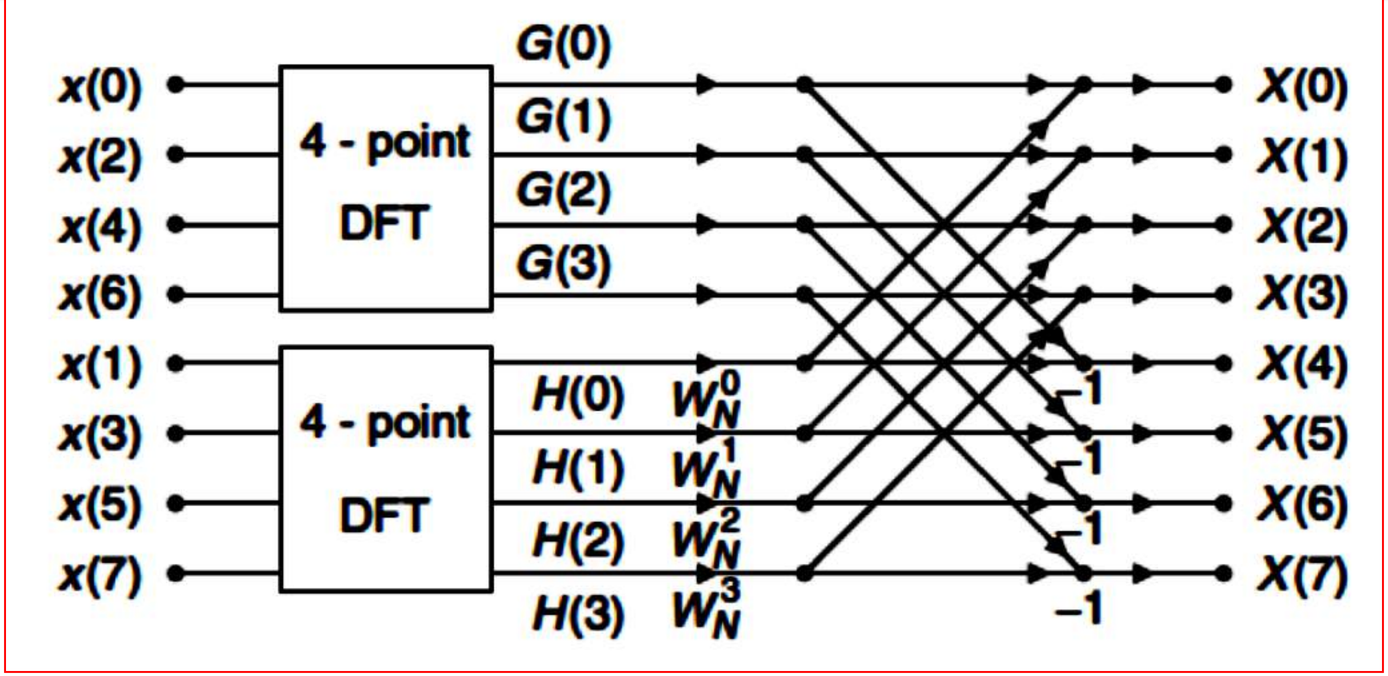
$$H(k) = H\left(k + \frac{N}{2}\right), \text{ for } k = 0, 1, \ldots, \frac{N}{2} - 1.$$

$$X(k) = G(k) + W_N^k H(k), \text{ for } k = 0, 1, \ldots, \frac{N}{2} - 1.$$

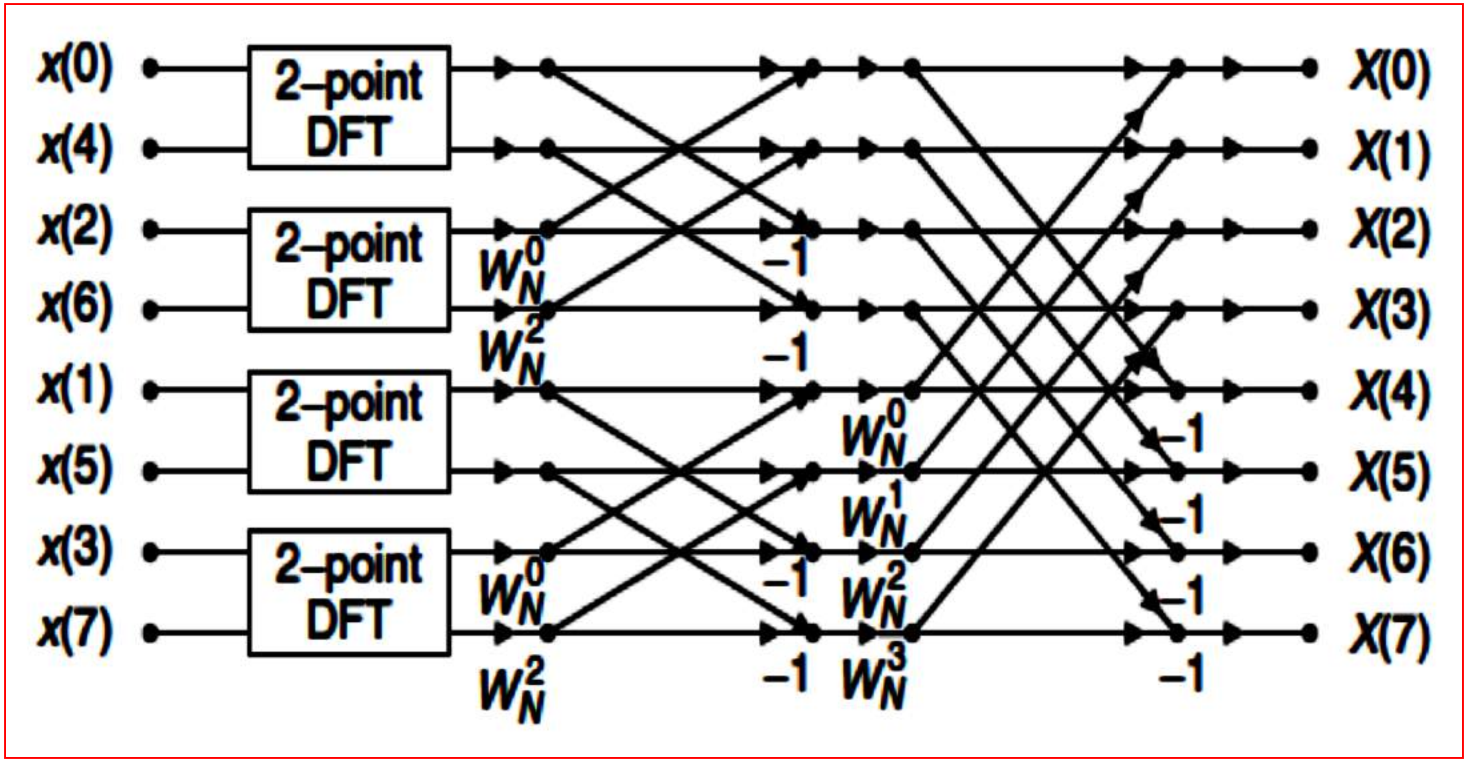
$$W_N^{(N/2+k)} = -W_N^k.$$

$$X\left(\frac{N}{2}+k\right) = G(k) - W_N^k H(k), \text{ for } k = 0, 1, \ldots, \frac{N}{2} - 1.$$

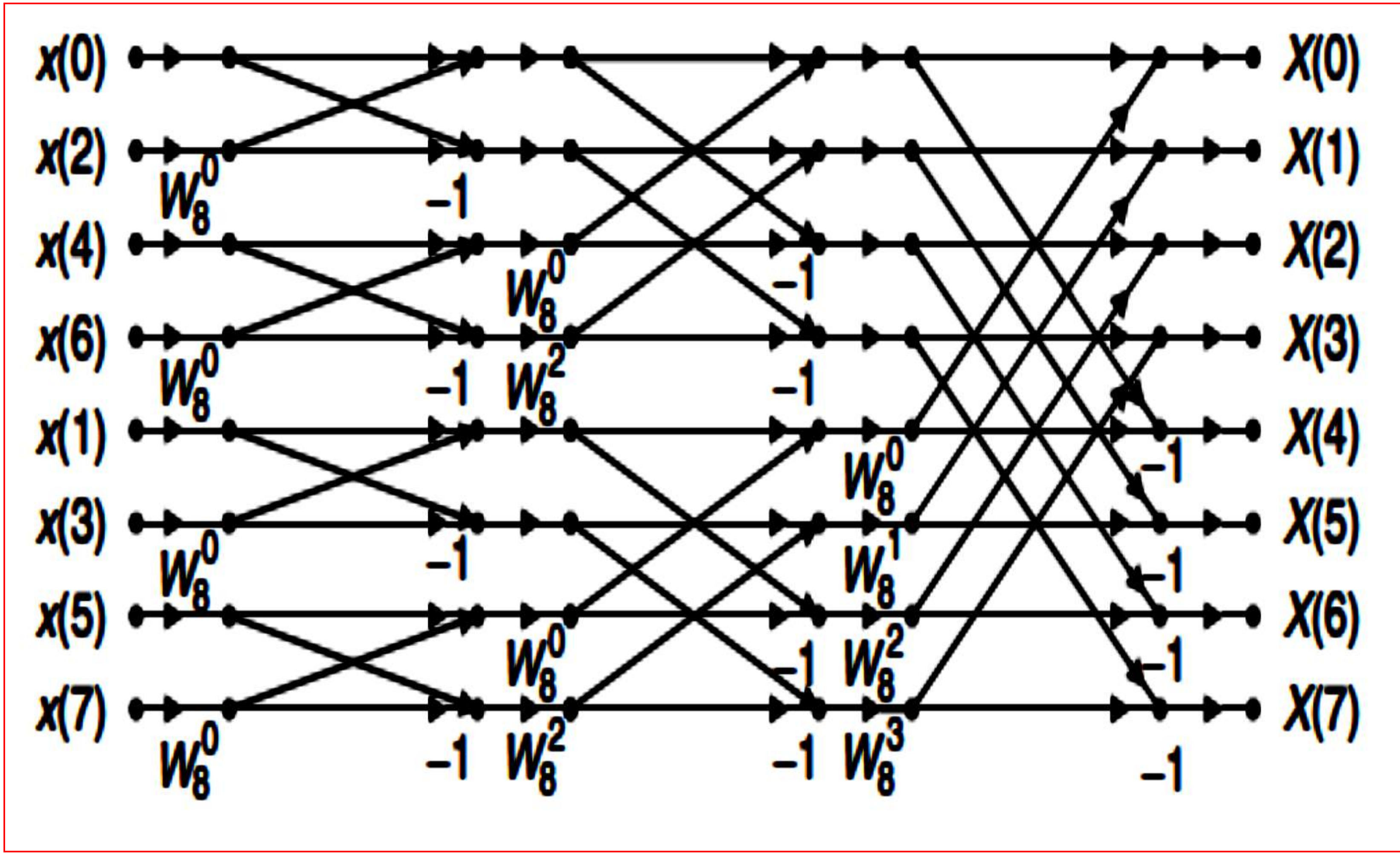
If we perform backward iterations, we can obtain the FFT algorithm. The procedure is illustrated in the figure below, the block diagram for the eight-point FFT algorithm.



From a further iteration, we obtain the figure shown below.



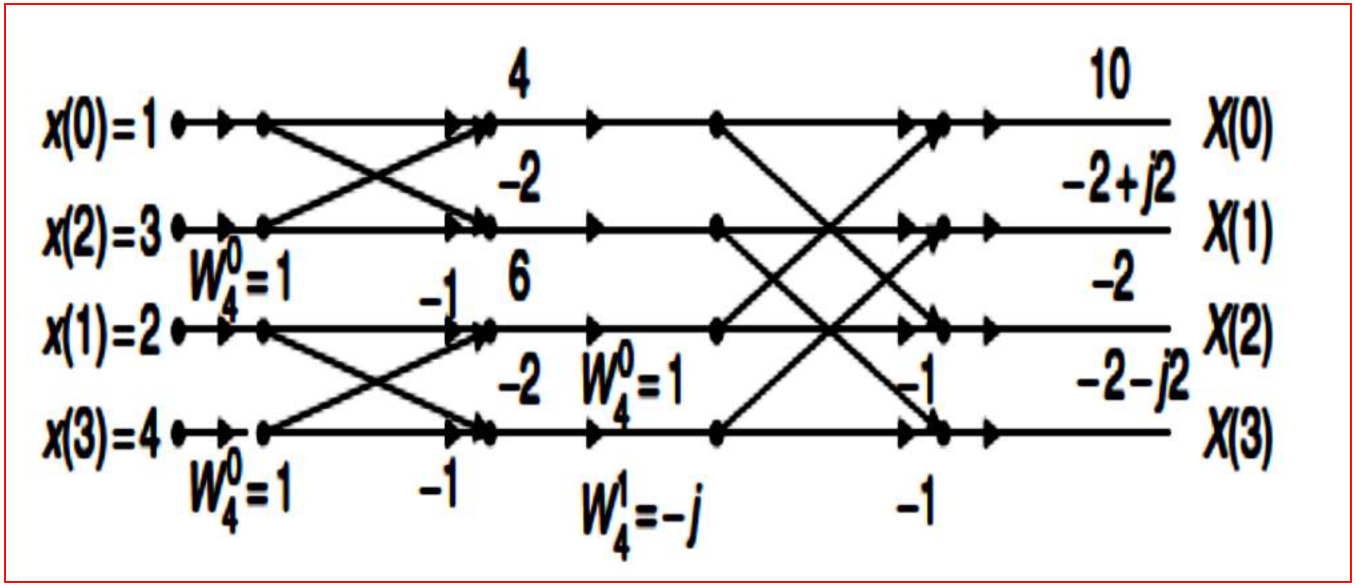
Finally, after three recursions, we end up with the block diagram in the figure below.



Example

Given a sequence *x(n)* for $0 \le n \le 3$, where x(0) = 1, x(1) = 2, x(2) = 3, and x(3) = 4, Evaluate its DFT X(k) using the decimation-in-time FFT method.

Solution:



Example

Given the DFT sequence $X(k)$ for $0 \leq k \leq 3$ computed in previous example, evaluate its inverse DFT $x(n)$ using the decimation-in-time FFT method.

Solution: