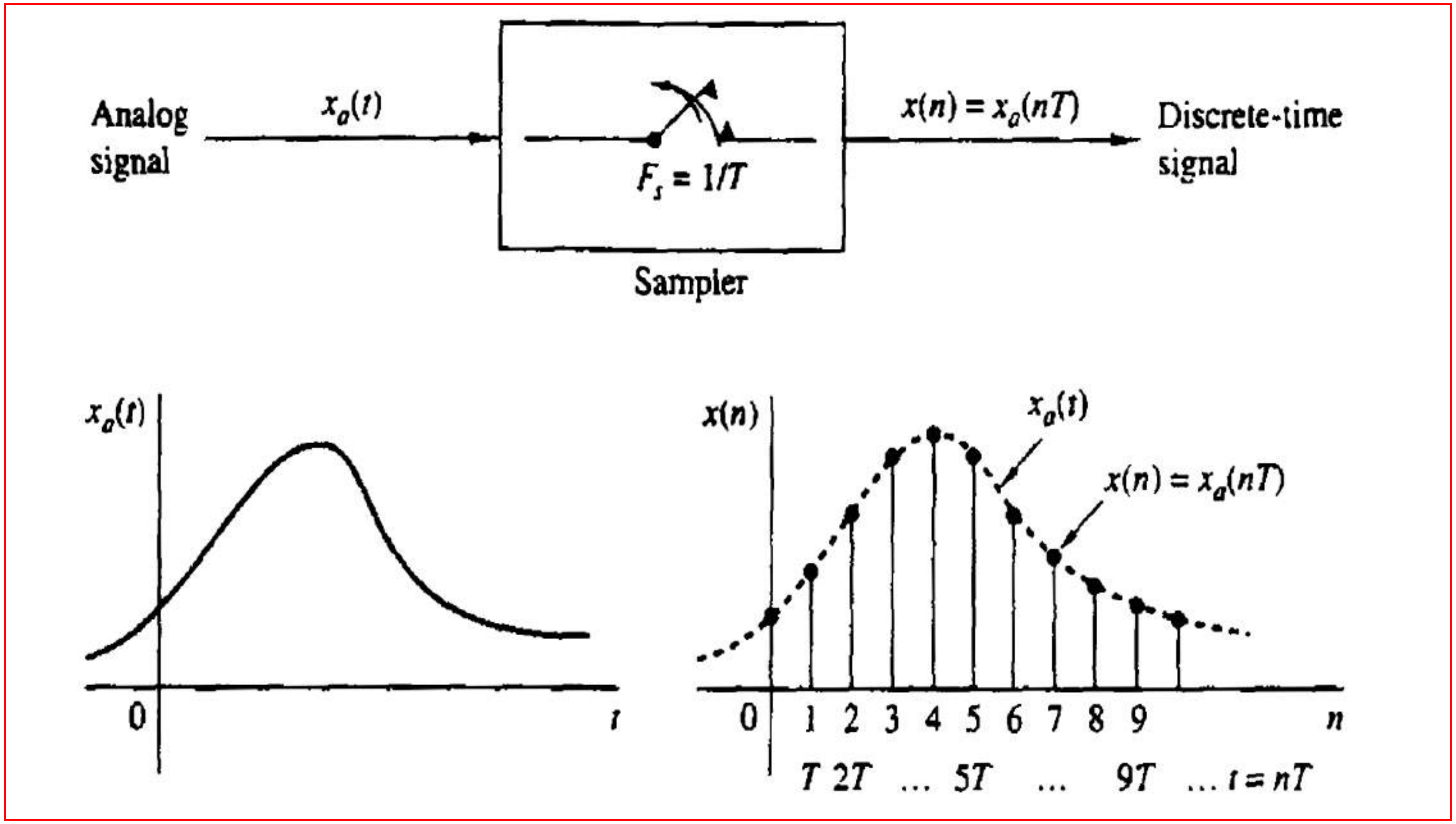
Sampling of Analog Signals



The variables t and n are linearly related through the sampling period T or, equivalently, through the sampling rate $F_s = 1/T$, as

$$t = nT = \frac{n}{F_s}$$

As a consequence of above equation, there exists a relationship between the frequency variable F (or Ω) for analog signals and the frequency variable f (or ω) for discrete – time signals. To establish this relationship, consider an analog sinusoidal signal of the form:

$$x_a(t) = A\cos(2\pi F t + \theta)$$

$$x_a(nT) \equiv x(n) = A\cos(2\pi F n T + \theta)$$

$$= A\cos\left(\frac{2\pi n F}{F_s} + \theta\right)$$

Continuous – time signals	Discrete – time signals
$\Omega = 2\pi F$	$\omega = 2\pi f$
$\frac{radians}{sec}$ Hz	$\frac{radians}{sample}$ $\frac{cycles}{sample}$

$$\omega = \Omega T,\ f = F/F_s$$ (→)

$$\Omega = \omega/T,\ F = f.F_s$$ (←)

Continuous – time signals	Discrete – time signals
$-\infty < \Omega < \infty$	$-\pi/T \le \omega \le \pi/T$
$-\infty < F < \infty$	$-F_s/2 \le f \le F_s/2$

From these relations we observe that

$$f_{max} = \frac{F_s}{2} = \frac{1}{2T}$$

$$\omega_{max} = \pi F_s = \frac{\pi}{T}$$

The sampling Theorem

If the highest frequency contained in an analog signal $x_a(t)$ is $F_{max} = B$ and the signal is sampled at a rate $F_s > 2F_{max} \equiv 2B$ (Nyquist rate).

Example Consider the analog signal

$$x_a(t) = 3cos50\pi t + 10sin300\pi t - cos100\pi t$$

What is the Nyquist rate for this signal?

Solution The frequencies present in the signal above are

$F_1 = 25$ Hz $\quad F_2 = 150$ Hz $\quad F_3 = 50$ Hz

Thus $F_{max} = 150$ Hz and $F_s > 2F_{max} = 300$ Hz $= F_N$

Example Consider the analog signal

$$x_a = 3cos2000\pi t + 5sin6000\pi t + 10cos12000\pi t$$

a) What is the Nyquist rate for this signal?

b) Assume now that we sample this signal using a sampling rate $F_s = 5000$ samples/sec. what is the discrete – time signal obtained after sampling?

c) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Solution

a) The frequencies existing in the analog signal are

F_1 = 1KHz, F_2 = 3 KHz, F_3 = 6 KHz

Thus F_{max} = 6 KHz, and according to the sampling theorem

$F_s > 2 F_{max}$ = 12 KHz and the Nyquist rate is F_N = 12 KHz

b) Since we have chosen F_s = 5 KHz, the folding frequency is

$$\frac{F_s}{2} = 2.5\, KHz$$

And this is the maximum frequency that can be represented uniquely by the sampled signal. We obtain

$$x(n) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$

$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(\frac{3}{5}\right)n + 10cos2\pi\left(\frac{6}{5}\right)n$$

$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(1-\frac{2}{5}\right)n + 10cos2\pi\left(1+\frac{1}{5}\right)n$$

$$= 3cos2\pi\left(\frac{1}{5}\right)n + 5sin2\pi\left(-\frac{2}{5}\right)n + 10cos2\pi\left(\frac{1}{5}\right)n$$

Finally, we obtain

$$x(n) = 13cos2\pi\left(\frac{1}{5}\right)n - 5sin2\pi\left(\frac{2}{5}\right)n$$

c) Since only the frequency components at 1 KHz and 2 KHz are present in the sampled signal, the analog signal we can recover is

$$y_a(t) = 13cos2000\pi t - 5sin4000\pi t$$

Quantization of Continuous – Amplitude signal

The process of converting a discrete – time continuous – amplitude signal into a digital signal by expressing each sample value as a finite (instead of an infinite) number of digits, is called quantization. The error introduced in representing the continuous – valued signal by a finite set of discrete value levels is called quantization error or quantization noise.

We denote the quantizer operation on the samples x(n) as Q[x(n)] and let $x_q(n)$ denote the sequence of quantized samples at the output of the quantizer. Hence

$$x_q(n) = Q[x(n)]$$

Then the quantization error is a sequence e_q (n) defined as the difference between the quantized value and the actual sample value. Thus

$$e_q(n) = x_q(n) - x(n)$$

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A unipolar quantizer deals with analog signals ranging from 0 volt to a positive reference voltage, and a bipolar quantizer has an analog signal range from a negative reference to a positive reference. The notations and general rules for quantization are:

$$\Delta = \frac{(x_{\max} - x_{\min})}{L}$$

$$L = 2^m$$

$$i = round\left(\frac{x - x_{\min}}{\Delta}\right)$$

$$x_q = x_{\min} + i\Delta, \text{ for } i = 0, 1, \ldots, L-1$$

Where:

X_{max} **and** X_{min}**:** are the maximum and minimum values, respectively, of the analog input signal x.

L: denotes the number of quantization levels.

m: is the number of bits used in ADC.

Δ: `is the step size of the quantizer or the ADC resolution.

Xq: indicates the quantization level.

i: is an index corresponding to the binary code.

Example:

Assuming that a 3-bit ADC channel accepts analog input ranging from 0 to 5volts, determine the following:

a. Number of quantization levels.

b. Step size of the quantizer.

c. Quantization level when the analog voltage is 3.2 volts.

d. Binary code produced by the ADC.

Solution:

a- $no.of\ quantization\ levels = 2^{no.of\ bits}$

$$no.of\ quantization\ levels = 2^3 = 8\ levels$$

b- Since the range is from 0 to 5 volts and the 3-bit ADC is used, we have X_{min}= 0 volt, X_{max}= 5 volts, and m= 3 bits.

$$\Delta = \frac{5-0}{8} = 0.625 \text{ volt.}$$

c- We determine quantization level when the analog voltage is 3.2 volts.

$$i = round\left(\frac{x - x_{\min}}{\Delta}\right) = round(5.12) = 5.$$

$$x_q = 0 + 5\Delta = 5 \times 0.625 = 3.125 \text{ volts.}$$

d- The binary code is determined as 101.